Normal default rules as epistemic actions

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Abstract

The goal of this paper is to present a prospective way to 'translate' normal default rules into the framework of action models logic. At the beginning we introduce default logic and normal default logic with their main properties and, separately, action models logic. Then a 'translation' of normal default rules in a slightly modified action models logic is presented.

1 Reasoning with default rules

Using the word *reasoning* we mostly mean 'private' act of an subject.[1] Common reasoning that we do all the time is often based on two kinds of information. The first one, *hard information*, is information that we are obliged to accept; we are sure of it, it is our knowledge and trusted data. The other one, *soft information*, is something that 'typically happens', it is very likely that things go that way. Reasoning based exclusively on hard information would be ideally deductive. However, it is not possible in common reasoning. There are many typical situations that are produced by our experience. Some of them are in contradiction, some of them are incomplete. Nonetheless, we have to do conclusions even if there is a lack of hard information. Such conclusions can be out of the scope of classical (deductive) consequence relations.

Let us imagine that we know that Anne is a student of a faculty of arts. A typical student from a faculty of arts does not like mathematics and we could conclude by default that Anne does not like mathematics. The knowledge that Anne attends a faculty of arts together with our prejudice that students from this faculty do not like math can form a (default) 'rule':

> If *AnneStudentOfArtFaculty*, then $\neg$*AnneLikeMath* under condition that the conclusion $\neg$*AnneLikeMath* is not in a conflict with our current knowledge.

Later we obtain an information that Anne studies logic. Well, if she studies logic, she might like mathematics. Similarly with this information, we can formulate a 'rule':

[1] Let us call a 'reasoning subject' an *agent*. The word 'private' will be discussed later on.

> If *AnneStudentOfLogic*, then *AnneLikeMath* under condition that the conclusion *AnneLikeMath* is not in a conflict with our current knowledge.

More formally, if we put hard information together, we obtain a set (of facts)

$$\Gamma = \{\text{AnneStudentOfArtFaculty, AnneStudentOfLogic}\}$$

and our reasoning about math's popularity of Anne can follow rules like these:[2]

$$\frac{\text{AnneStudentOfArtFaculty} \ : \ \neg\text{AnneLikeMath}}{\neg\text{AnneLikeMath}}$$

$$\frac{\text{AnneStudentOfLogic} \ : \ \text{AnneLikeMath}}{\text{AnneLikeMath}}$$

Note that once we deduce that Anne does not like mathematics we may not use the second rule because its presupposition is in conflict with our current knowledge, i.e., with the conclusion that Anne does not like math.[3]

In real life, one might introduce a preference relation between these two rules. For example, there is a study program of Logic at the Faculty of Arts in Prague and every student of logic is also a student of this faculty. Therefore the fact that Anne studies logic is more informative here than the fact that she studies at the faculty. One might thus prefer to use the second rule since its assumption is 'more informative'.

The idea of common reasoning formalization is mostly studied in non-monotonic logic—logical systems where monotony can fail. A conclusion of a set of premises needs not be a conclusion anymore if we extend the set of premises.[4] In this paper we will use a formal system called *default logic* with operational semantics; cf. [1], especially. Our aim is to introduce the idea of 'translating' default rules in action model logic. For these purposes we will introduce only the plain version of default logic. We will have a set of premises (hard information) together with a set of default rules that form a default theory.

1.1 Operational semantics

A (general) *default theory* T is a couple (Γ, D) where the set of formulas Γ represents 'hard information', which is accepted as true, and D is a finite set of *default rules* (or *defaults*, for short). A default (rule) can be understood as an accepted way to extend our 'hard information'. It enables us to do conclusions that extend possibilities of classical consequence relation.

A (general) default rule is of the form

$$\frac{\varphi : \psi_1, \psi_2, \ldots, \psi_n}{\chi}$$

[2]The rule is in the form of a fraction; hard information is written before the colon together with what is presupposed (behind the colon), a consequent is under the line.

[3]If we conclude that Anne does not like mathematics, we cannot consistently assume that she does like mathematics (and vice versa).

[4]See, e.g., [1], [2], [3], and [5].

where φ, $\psi_1, \ldots, \psi_n$, and χ are formulas of a background logic.[5] The meaning of a general default can be:

> If (a *prerequisite*) φ is 'known' to be true and (*justifications*) $\psi_1, \ldots, \psi_n$ can be consistently presupposed, then (a *consequent*) χ is derivable.[6]

There is an idea presented in [1] on how to work with defaults algorithmically. Let us imagine we form sequences of defaults from the set D without multiple occurrence: Π_1, Π_2, Π_3, etc. The ordering of defaults in $\Pi_j = \langle d_{j_1}, \ldots, d_{j_k} \rangle$, where $\{d_{j_1}, \ldots, d_{j_k}\} \subseteq D$, is an order of their possible applicability. Before we introduce the term *applicability* of a default rule to a deductively closed set of formulas, we define two auxiliary sets, $\mathsf{In}\Pi_j$ and $\mathsf{Out}\Pi_j$, for each sequence Π_j. Both sets must be understood as arising step by step, i.e., default by default, according to an order in Π_j. The definition is by recursion, where Π_j^m denotes the initial segment of $\Pi_j = \langle d_{j_1}, \ldots, d_{j_k} \rangle$ of length m, where $m \leq k$:

$$
\begin{aligned}
\mathsf{In}\Pi_j^0 &= \mathsf{Cn}(\Gamma) \\
\mathsf{In}\Pi_j^{m+1} &= \mathsf{Cn}\left(\mathsf{In}\Pi_j^m \cup \left\{\chi \mid \tfrac{\varphi:\psi_1,\psi_2,\ldots,\psi_n}{\chi} = d_{j_{m+1}}\right\}\right) \\
\mathsf{In}\Pi_j &= \mathsf{In}\Pi_j^k \\
\mathsf{Out}\Pi_j^0 &= \emptyset \\
\mathsf{Out}\Pi_j^{m+1} &= \mathsf{Out}\Pi_j^m \cup \left\{\neg\psi_1, \ldots, \neg\psi_n \mid \tfrac{\varphi:\psi_1,\psi_2,\ldots,\psi_n}{\chi} = d_{j_{m+1}}\right\} \\
\mathsf{Out}\Pi_j &= \mathsf{Out}\Pi_j^k
\end{aligned}
$$

Step 0 does not apply any default rule, but it prepares all what is (logically) obtainable from the 'hard information' Γ. In step 1, we take the first default in a sequence Π_j and test its applicability (with respect to $\mathsf{In}\Pi_j^0$):[7] φ is included in (the so far obtained) $\mathsf{In}\Pi_j^0$ and no $\psi_1, \ldots, \psi_n$ is in contradiction with (the so far obtained) $\mathsf{In}\Pi_j^0$, i.e., $\neg\psi_l \notin \mathsf{In}\Pi_j^0$, for each $l \in \{1, \ldots, n\}$. If the default is applicable, sets $\mathsf{In}\Pi_j^1$ and $\mathsf{Out}\Pi_j^1$ are created

$$
\begin{aligned}
\mathsf{In}\Pi_j^1 &= \mathsf{Cn}\left(\mathsf{In}\Pi_j^0 \cup \left\{\chi \mid \tfrac{\varphi:\psi_1,\psi_2,\ldots,\psi_n}{\chi} = d_{j_1}\right\}\right) \\
\mathsf{Out}\Pi_j^1 &= \left\{\neg\psi_1, \ldots, \neg\psi_n \mid \tfrac{\varphi:\psi_1,\psi_2,\ldots,\psi_n}{\chi} = d_{j_1}\right\}
\end{aligned}
$$

and we can continue with step 2, and so on.

Definition 1. *A default* $\frac{\varphi:\psi_1,\psi_2,\ldots,\psi_n}{\chi}$ *is* applicable *to a deductively closed set* Δ *iff* $\varphi \in \Delta$ *and* $\neg\psi_1 \notin \Delta, \ldots, \neg\psi_n \notin \Delta$.

Our introductory example can be formalized as a default theory where $\Gamma = \{\varphi, \psi\}$ and $D = \left\{\frac{\varphi:\neg\chi}{\neg\chi}, \frac{\psi:\chi}{\chi}\right\}$. We can form five sequences of defaults from D:

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[5] In this paper, we use classical propositional language (modalities will be added later on). Consequence relation (resp. operation Cn) is based on classical propositional logic. A set of formulas Δ is deductively closed iff $\Delta = \mathsf{Cn}\Delta$.

[6] There are always at least one justification and consequent. It is possible to have no prerequisite. In that case we interpret the empty 'prerequisite place' as *true*, i.e., tautology.

[7] For simplicity, let us assume that $d_{j_1} = \frac{\varphi:\psi_1,\psi_2,\ldots,\psi_n}{\chi}$.

$$\Pi_1 = \langle\ \rangle \qquad \Pi_2 = \left\langle \frac{\varphi:\neg\chi}{\neg\chi} \right\rangle \qquad \Pi_3 = \left\langle \frac{\psi:\chi}{\chi} \right\rangle$$
$$\Pi_4 = \left\langle \frac{\varphi:\neg\chi}{\neg\chi}, \frac{\psi:\chi}{\chi} \right\rangle \quad \Pi_5 = \left\langle \frac{\psi:\chi}{\chi}, \frac{\varphi:\neg\chi}{\neg\chi} \right\rangle$$

Let us look at Π_5, for example. The first default is applicable to $\mathsf{Cn}\Gamma$ since $\psi \in \mathsf{Cn}\Gamma$ and $\neg\chi \notin \mathsf{Cn}\Gamma$. But the second default is not applicable now. Formula χ is derivable from $\mathsf{Cn}(\mathsf{Cn}\Gamma \cup \{\chi\})$.

Definition 2 (Process)**.** *A sequence of default rules* Π *is a* process *of a default theory* T *iff the default* d_m *is applicable to* $\mathsf{In}\Pi^m$*, for every* m *such that* $d_m \in \Pi$.[8]

In the example, Π_1, Π_2, and Π_3 are processes.

Definition 3. *Let* Π *be a process.*

- Π *is* successful *iff* $\mathsf{In}\Pi \cap \mathsf{Out}\Pi = \emptyset$*, otherwise it is* failed*.*
- Π *is* closed *iff every* $d \in D$ *applicable to* $\mathsf{In}\Pi$ *(i.e., in the order) is already in* Π*.*

Processes Π_1, Π_2, and Π_3, from the example, are successful; however, Π_1 is not closed since there is at least one default in D that is applicable. Generally, it can happen that there is an applicable default in a process whose consequent causes the fail. For example, consider a 'strange' theory $\left(\Gamma = \emptyset, D = \left\{\frac{:p}{\neg p}\right\}\right)$, where p is an atomic formula. It has an applicable rule with the consequent $\neg p$, which is in a conflict with the justification p.

Definition 4 (Extension)**.** *A set of formulas* E *is an* extension *of* T *iff there is a successful and closed process* Π *such that* $E = \mathsf{In}\Pi$*.*

Extension is a central notion of default logic. It is a deductively closed set containing conclusions of hard information together with consequents of applied defaults. Moreover, we are sure that there is not any applicable default left. As it is seen in our introductory example, there can be more than one extension. Π_2 and Π_3 are successful and closed processes that form two different extensions. Every extension can be understood as a way to extend deduction over hard information consistently.

The most important properties of (general) default theories are:[9]

1. Let E_1, E_2 be extensions of T and $E_1 \subseteq E_2$, then $E_1 = E_2$ (*minimality of extensions*).
2. $T = (\Gamma, D)$ has an inconsistent extension if and only if Γ is inconsistent itself (*consistency preservation*).
3. Let E be an extension of $T = (\Gamma, D)$, then E is an extension of $T' = (\Gamma \cup \Delta, D)$ for every $\Delta \subseteq E$ (*cautious monotony in declaratives*).

[8] We will omit the index j in Π_j whenever it is not necessary to distinguish various default sequences of one default theory.

[9] Proofs are easy for finite sets of defaults. See [1, pp. 42–44].

Minimality of extensions (1) says that if there are two different extensions, then they must be incompatible. Consistency preservation (2) guarantees that applications of defaults does not produce inconsistency. As a corollary of this property we obtain: If T has an inconsistent extension E, then E is its only extension.[10] And if $T = (\Gamma, D)$ has two different extensions, then Γ is consistent. The last property (3) is a form of monotonicity; we can add hard information, which is based on an extension, and it does not cause any change in 'conclusions' (extensions).

1.2 Normal default theories

There is a special class of defaults called normal default rules. These rules have the general form of

$$\frac{\varphi : \psi}{\psi}$$

Call a default theory normal if it contains only normal default rules. The introductory example is formalized as a normal default theory. Normal defaults cannot cover the full range of non-monotonic reasoning, but they can formalize much from the common reasoning. Above that, normal default theories have many desirable properties. One of the most important is that normal theories have always at least one extension.[11]

Proposition 1 (Existence of extensions)**.** *If $T = (\Gamma, D)$ is a normal default theory, then T has an extension.*

Proof. We will reason that normal theories have always at least one closed and successful process. The case of inconsistent theories and their extensions is clear. Let us consider consistent ones.

First, every process of a default theory T can be extended to a closed process. This is true for general default theories. If there is a process Π and a default $d \in D$, which is applicable, check whether d is in Π. If not, add it. And so on. (For infinite process, see the proof in [1, pp. 33–34].)

Second, if Π is a process of a normal default theory T, then Π is successful. This follows from the form of normal default rules. The applicability check is processed just on the formula that is in the role of the consequent [1, p. 50]. $\square$

Normal default theories are monotonic in sets of defaults. If we add a new default, then we 'only extend' the original extensions [1, p. 50].

Proposition 2 (Monotony in defaults)**.** *Let $T_1 = (\Gamma, D_1)$ and $T_2 = (\Gamma, D_2)$ be normal default theories such that $D_1 \subseteq D_2$. Then each extension E_1 of T_1 is a subset of some extension E_2 of T_2, i.e., $E_1 \subseteq E_2$.*

[10] Such extension is just the set of all formulas.

[11] We have mentioned the (non-normal) theory $\left(\emptyset, \left\{\frac{:p}{\neg p}\right\}\right)$ that has no extensions. It has two processes. The empty process $\langle\ \rangle$ is not closed. The process $\left\langle \frac{:p}{\neg p} \right\rangle$ is closed, but not successful.

In our example with Anne studying logic and faculty of arts, we obtained two different extensions that are incompatible, i.e., they are inconsistent together. This is an inherent property of normal theories [1, p. 52].

Proposition 3 (Orthogonality of extensions)**.** *If a normal default theory T has two different extensions E_1 and E_2, then $E_1 \cup E_2$ is inconsistent.*

All the mentioned properties of normal default theories can be considered as reasonable for common 'private' reasoning of an agent. Here we accept the idea that extensions play the role of conclusions, which can be derived from both hard and soft information and provide a support for decisions. The first property (*existence of extensions*) coincides with the model of an agent that is obliged to do conclusions and decisions. If there are different conclusions, it means that there are different and incompatible ways of doing decisions (*orthogonality of extensions*). If an agent accepts new soft information, it can extend a decision support without destroying it (*monotony in defaults*).

2 Action models

The system of action models we will be using, was published in [7] as a way to describe and formalize epistemic actions. Action model logic is a variant of dynamic logic. Generally speaking, an action formalizes a transition from one epistemic state to (another) epistemic state. For the modelling of epistemic states we use standard propositional *epistemic logic*.[12]

Epistemic logic is a multimodal system extending the classical propositional logic. The language contains a set of atomic formulas $\mathcal{P}$, a finite set of agents $\mathcal{A}$, and formulas defined by BNF:

$$\psi ::= p \mid \neg\psi \mid \psi \to \psi \mid K_i\psi \mid \hat{K}_i\psi$$

where $i \in \mathcal{A}$ can be interpreted as a name of an agent. We use the well-known $\mathsf{S5}$ semantics. Kripke model is a structure $M = (W, R_i, V)$ where W is a non-empty set of possible worlds, R_i is an accessibility relation of an agent $i \in \mathcal{A}$, and V is a valuation function.[13]

The satisfaction relation $\Vdash$ is defined in a standard way:

- $(M, w) \Vdash p$ iff $w \in V(p)$
- $(M, w) \Vdash \neg\psi$ iff $(M, w) \nVdash \psi$
- $(M, w) \Vdash \psi_1 \to \psi_2$ iff $(M, w) \Vdash \psi_1$ implies $(M, w) \Vdash \psi_2$
- $(M, w) \Vdash K_i\psi$ iff $(M, v) \Vdash \psi$, for each v such that wR_iv

[12] See, e.g., [7] and [6].

[13] For simplicity we will use the single-agent setting throughout the paper, except some comments in subsection 3.1.

Modality K_i represents knowledge of an agent i and modality $\hat{K}_i$ is understood as a dual to K_i:

$$\hat{K}_i\psi \equiv \neg K_i \neg \psi$$

To obtain an action-model version of epistemic logic, we extend the epistemic language by actions that represent the transformation of one epistemic model to another one. The epistemic language will be extended by new modalities $[\alpha]$ and $\langle\alpha\rangle$ where α is an *action*.[14] The semantics is enriched by the following clauses:

- $(M, w) \Vdash [\alpha]\psi$ iff $(M, w) \xrightarrow{\alpha} (M', w')$ implies $(M', w') \Vdash \psi$, for all (M', w')
- $(M, w) \Vdash \langle\alpha\rangle\psi$ iff $(M, w) \xrightarrow{\alpha} (M', w')$ and $(M', w') \Vdash \psi$, for some (M', w')

Let us note that $\langle\alpha\rangle$ is dual to $[\alpha]$. Now we are obliged to explain how to understand the application of an α-action on an epistemic state (M, w), i.e., what is the meaning of $(M, w) \xrightarrow{\alpha} (M', w')$ in *action model logic.*

An action α causes a change of an epistemic state (M, w) to a state (M', w'). This change is conducted by structures (called *action models*) that are very similar to Kripke structures that we use as models for epistemic logic. An action α can be atomic or composite.[15] Atomic actions and composite actions as well are based on action models. An *action model* is a structure

$$\mathbf{M} = (\mathbf{S}, \mathbf{R}_i, \mathsf{pre})$$

where

- $\mathbf{S}$ is a non-empty set of nodes
- $\mathbf{R}_i$ is a binary relation on $\mathbf{S}$, i.e., $\mathbf{R}_i \subseteq \mathbf{S} \times \mathbf{S}$ for each $i \in \mathcal{A}$
- pre is a function assigning exactly one formula to each node ($\mathsf{pre} : \mathbf{S} \longmapsto \mathsf{Fla}$).

Action models have non-empty domains of action-states (nodes) and an accessibility relation for each agent. These relations have the same constrictions and properties as their epistemic counterparts, i.e., since our underlying logic is $\mathsf{S5}$, the relations are reflexive, transitive, and symmetric. The informal interpretation is the same as in plain $\mathsf{S5}$ system, $\mathsf{s}\mathbf{R}_i\mathsf{t}$ means that an agent i cannot distinguish action-states s and t. Unlike Kripke models however, action models do not contain a binary relation between nodes and formulas (valuation). They instead have a unary function, called *precondition*, that assigns a formula to every node of an action model. This precondition formula has to be satisfied in order for the respective action to happen.

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[14] The form of actions will be discussed immediately. For the moment, the reader can imagine that α is a (computer) program as in dynamic logic [4].

[15] E.g., for any actions α_1 and α_2, concatenation of two actions $\alpha = (\alpha_1; \alpha_2)$, finite repetition of an action $\alpha = (\alpha_1)^*$, non-deterministic choice of two actions $\alpha = (\alpha_1 \sqcup \alpha_2)$, and others.

Given an epistemic model, one may apply an action model to it. This 'action execution' is governed by a function of restricted modal product $\otimes$ that takes an epistemic model and an action model and creates a new epistemic model.

Definition 5 (Restricted modal product)**.** *Let* $M = (W, R_i, V)$ *be an* $\mathsf{S5}$ *epistemic model and* $\mathbf{M} = (\mathbf{S}, \mathbf{R}_i, \mathsf{pre})$ *an* $\mathsf{S5}$ *action model. A* restricted modal product $(M \otimes \mathbf{M})$ *is an epistemic model* $M' = (W', R'_i, V')$ *where*

- $W' = \{(w, \mathsf{s}) \mid w \in W \;\&\; \mathsf{s} \in \mathbf{S} \;\&\; (M, w) \Vdash \mathsf{pre}(\mathsf{s})\}$
- $(w, \mathsf{s})R'_i(w', \mathsf{s}')$ *iff* $(wR_iw' \;\&\; \mathsf{s}\mathbf{R}_i\mathsf{s}')$*,*
  for $w, w' \in W$ *and* $\mathsf{s}, \mathsf{s}' \in \mathbf{S}$
- $(w, \mathsf{s}) \in V'(p)$ *iff* $w \in V(p)$*,*
  for $(w, \mathsf{s}) \in W'$ *and atomic formula* $p \in \mathcal{P}$

Thus, in action model logic, the change from an epistemic state (M, w) to a 'new' epistemic state (M', w') can be conducted by an atomic action $\alpha = (\mathbf{M}, \mathsf{s})$:

$$(M, w) \overset{(\mathbf{M},\mathsf{s})}{\longrightarrow} (M', w')$$

if and only if

$$w \Vdash \mathsf{pre}(\mathsf{s}) \text{ and } (M', w') = ((M \otimes \mathbf{M}), (w, \mathsf{s}))$$

The language of action model logic includes epistemic language and atomic actions that are always of the form $(\mathbf{M}, \mathsf{s})$. Composite actions are usually reducible to atomic ones. More about the properties of this system can be found in [7, chapter 6].

3 Normal default actions

In action model logic we work with the idea that there is a group of agents and these agents change their epistemic states with respect to 'new' information produced by verbal as well as non-verbal actions. There is a slightly modified picture in the introductory example. We played the role of an agent reasoning about 'Anne's popularity of math'. We called this reasoning 'private' because it mostly happens inside our head without any connection with other agents. Nonetheless, it need not be completely private, other agents can know both hard and soft information and can follow our steps in reasoning. Even if we do not show publicly the direction of our thoughts, the other agents have to consider all possibilities in their epistemic states.

To incorporate default reasoning inside action model logic we have to understand default rules as actions.[16] Someone (let us call the agent i) can use the default rule

$$\frac{\text{AnneStudentOfArtFaculty} \;:\; \neg\text{AnneLikeMath}}{\neg\text{AnneLikeMath}}$$

[16]Defaults cause changes, which exceed the deductive base of a background formal system that is described by epistemic logic.

in a situation whenever i is not sure whether Anne likes math or not, but there is no information, which is in conflict with the justification that *Anne does not like math.* Simultaneously, the prerequisite *Anne is a student of a faculty of arts* is considered by i as a 'knowledge'. If the rule is applied, then i narrows down the set of possible worlds that are indistinguishable for i. Now, the agent accepts *Anne does not like math* as a (new) 'knowledge'. In fact, if a normal default rule is applied by i, then i changes her epistemic situation that forms a base for a possible application of other defaults. More formally, a normal default rule $\frac{\varphi:\psi}{\psi}$ changes agent's epistemic model such that it separates ψ-worlds from $\neg\psi$-worlds and the agent is ready to work with preferred ψ-worlds from now on. If the agent i goes on with some other default, she only checks the validity of a (new) prerequisite with respect to the possible worlds where ψ is true. Of course, it does not mean that $\neg\psi$-worlds are canceled. They are now distinguishable for i and can be important from the viewpoint of other agents' knowledge.

This idea brings us to a small modification of epistemic models. Every agent will have a set of designated (preferred) possible worlds that are the basis of defaults' applicability.

Definition 6. *A (default) epistemic model M is a structure (W, R_i, V, X_i) where W is a non-empty set of possible worlds, R_i is an accessibility relation of an agent i, V is a valuation function, and $X_i \subseteq W$ is a non-empty set of designated possible worlds for an agent i such that whenever uR_iv, it holds that $u \in X_i$ iff $v \in X_i$.*

The designated set $X_i \neq \emptyset$ marks the states that are 'important' to agent i. In the agent's reasoning, i.e., in application of a default rule, the agent 'ignores' all the states outside of X_i. The designated worlds are not connected via R_i to those that are not designated.

This leads us to a formal solution of the question when a normal default rule $\frac{\varphi:\psi}{\psi}$ is *epistemic applicable* by an agent. An agent has to 'know' the prerequisite φ and consider the justification ψ as unknown, but possible, with respect to the set of the agent's designated worlds.

Definition 7. *A normal default rule $\frac{\varphi:\psi}{\psi}$ is* epistemic applicable *by an agent i in an epistemic model M iff for each $w \in X_i$:*

- $(M, w) \Vdash K_i\varphi$
- $(M, w) \Vdash \hat{K}_i\psi$
- $(M, w) \Vdash \hat{K}_i\neg\psi$

In other words, formulas $K_i\varphi$, $\hat{K}_i\psi$, and $\hat{K}_i\neg\psi$ are valid in the submodel of M generated by a set X_i.[17]

Our term of (epistemic) *applicability* does not fully correspond to the applicability in default logic. A default rule $\frac{\varphi:\psi}{\psi}$, where the justification ψ is known by an agent, could be applicable in default logic but from the epistemic point

[17]Compare the notion of 'R-applicability' in [5].

of view it does not do any change of agent's epistemic state. Such default rules would be in some sense hollow—empty thinking about things that are already known. However, we want defaults to decide unknown things.[18]

Actions based on normal default rules are of another nature than atomic actions in action models logic. They do not depend on one (action) node and its precondition. The applicability of a default takes over the role of precondition.

An action corresponding to a normal default $\frac{\varphi:\psi}{\psi}$ (used by an agent i) will be understood as a two-node action model

$$\mathbf{D}^i = (\mathbf{S}, \mathbf{R}_i, \mathsf{pre}, \mathbf{X}_i)$$

where

- $\mathbf{S} = \{\mathsf{s}, \mathsf{t}\}$
- $(\mathsf{s}, \mathsf{t}) \notin \mathbf{R}_i$, but $(\mathsf{s}, \mathsf{s}) \in \mathbf{R}_i$ and $(\mathsf{t}, \mathsf{t}) \in \mathbf{R}_i$
- $\mathsf{pre}(\mathsf{s}) = \psi$ and $\mathsf{pre}(\mathsf{t}) = \neg\psi$
- $\mathbf{X}_i = \{\mathsf{s}\}$

The action $\mathbf{D}^i$ has two action-states, which are not connected by the relation $\mathbf{R}_i$ (the agent i can distinguish these two states), and they differ in preconditions with respect to formula ψ. The new aspect is the set $\mathbf{X}_i$, which corresponds to the set of designated worlds in an epistemic model. $\mathbf{X}_i$ contains action states that have the formula ψ as their preconditions. In the simple case of normal defaults, the designated set $\mathbf{X}_i$ contains only one action-state s whose precondition is ψ.[19]

The idea will be complete after we describe how the action $\mathbf{D}^i$ works. We have emphasized that a normal default action is different from atomic actions in action model logic. It is not an update that changes one particular epistemic state (M, w) into a (new) epistemic state (M', w'). Normal default actions operate on whole models and, thus, change epistemic background.

For an epistemic model $M = (W, R_i, V, X_i)$ we define

$$M \xrightarrow{\mathbf{D}^i} M'$$

if and only if

1. the corresponding default rule is applicable (Definition 7) and
2. $M' = (M \otimes \mathbf{D}^i)$.

The resulting epistemic model M' will be formed as it is described in Definition 5. The only thing we have to add is how to form the new set of designated worlds X'_i. For each $w \in W$ and $\mathsf{x} \in \mathbf{S}$:

$$(w, \mathsf{x}) \in X'_i \text{ iff } w \in X_i \;\&\; \mathsf{x} \in \mathbf{X}_i$$

[18]Similarly, multiple occurrence of a default is not allowed in default logic.
[19]A two-agent version will be mentioned in subsection 3.1.

The action based on action model $\mathbf{D}^i$ causes the resulting epistemic model M' to contain two parts (submodels) that are disjoint for the accessibility relation R_i. Let us now consider normal defaults $\frac{\varphi:\psi}{\psi}$ and $\frac{\varphi:\neg\psi}{\neg\psi}$, for example. We will write $(\varphi : \psi/\psi)^i$ and $(\varphi : \neg\psi/\neg\psi)^i$ as actions based on these defaults for an agent i. Both of them are applicable (by the agent i) under the same conditions, cf. Definition 7.[20] The corresponding action models are almost the same. The difference lies in sets of designated worlds. The first default action model requires to have designated nodes with the precondition ψ and the other one with $\neg\psi$.

If the action $(\varphi : \psi/\psi)^i$ is applied on an epistemic model $M = (W, R_i, V, X_i)$ (by an agent i), then two separated parts are formed in the new epistemic model $M' = (W', R'_i, V', X'_i)$. No world from one part is connected by R'_i to any world from the other part. One of the parts consists of ψ-worlds and these worlds are designated (X'_i), the other one consists of $\neg\psi$-worlds. Informally, the agent i did a decision whether ψ or not and preferred ψ as true. If the agent i is in any new epistemic state $w' \in X'_i$, then the formula $K_i\psi$ is true there. Similarly for the formula $K_i\neg\psi$ in states out of X'_i.

In case we know that an agent i is in a particular epistemic state (M, w), then the application of either $(\varphi : \psi/\psi)^i$ or $(\varphi : \neg\psi/\neg\psi)^i$ depends on whether $(M, w) \Vdash \psi$ or $(M, w) \Vdash \neg\psi$. A particular epistemic state provides preferences among default actions based on preconditions.

From the viewpoint of an agent i and her epistemic model $M = (W, R_i, V, X_i)$, a default theory (Γ, D) means that $(M, w) \Vdash K_i\gamma$, for each $w \in X_i$ and each $\gamma \in \Gamma$. At the very beginning, before the use of any default, all states are designated ($X_i = W$). The final epistemic model is the result of 'step by step' applications of default actions given by a successful and closed process. If $\Pi = \langle d_1, \ldots, d_k \rangle$ is a successful and closed process, we obtain the final epistemic model (for an agent i) by the concatenation of corresponding actions $(\mathbf{D}^i_1; \mathbf{D}^i_2; \ldots; \mathbf{D}^i_k)$:

$$M \xrightarrow{\mathbf{D}^i_1} \cdots \xrightarrow{\mathbf{D}^i_k} M'$$

The knowledge of the agent i in a submodel (of M') generated by X'_i corresponds to the extension $\mathsf{In}\Pi$.[21]

Let us consider a normal default theory $\left(\emptyset, \left\{ \frac{:p}{p}, \frac{:\neg p}{\neg p}, \frac{p:q}{q} \right\}\right)$ of an agent i. There is no hard information and p, q are atomic formulas. We can form two successful and closed processes: $\left\langle \frac{:p}{p}, \frac{p:q}{q} \right\rangle$ and $\left\langle \frac{:\neg p}{\neg p} \right\rangle$. Thus, we have two possible updates of an epistemic model $M = (W, R_i, V, X_i)$. The first one is given by concatenation of two normal default actions $((: p/p)^i; (p : q/q)^i)$ and the second one by the single action $(: \neg p/\neg p)^i$. Now all depends on the applicability and agent's decision which one will be done.

[20] With respect to Definition 7 we can make a terminological convention. The meaning of 'a default rule $\frac{\varphi:\psi}{\psi}$ epistemic applicable by an agent i in an epistemic model' is the same as 'a default action $(\varphi : \psi/\psi)^i$ applicable (by i) in an epistemic model'.

[21] Formal details will not be discussed in this introductory paper.

If there is a concatenation of normal default actions, then the erasing of an accessibility relation is executed all over the model. For example, whenever i executes $((: p/p)^i; (p : q/q)^i)$, then, after the second action $(p : q/q)^i$, the relation R_i does not connect q- and $\neg q$-words in both parts formed by the first action $(: p/p)^i$. The final set of designated worlds will contain $(p \wedge q)$-worlds.

3.1 Examples

In formalisms and examples throughout the text we used, in fact, a single-agent variant. Our aim was to introduce the basic idea how to use normal defaults as actions. Nonetheless, it will be useful to present some notes concerning a multi-agent variant.

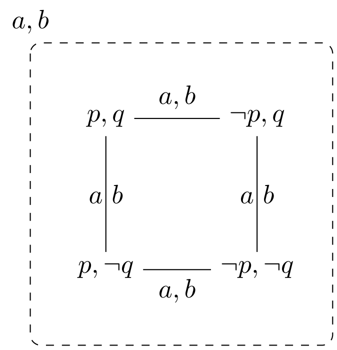
The language of epistemic logic, which we have introduced, is multi-agent friendly. Knowledge operators as well as accessibility relations are indexed by agents' names. Since default rules correspond to 'private' reasoning acts, we use the same indexing by the name of an agent for normal default actions. Nonetheless, we did not introduce group knowledge operators and that is the reason why we are not going to discuss all aspects of the multi-agent setting. We do not solve what is the essence of a default theory, whether 'hard information' is commonly known among agents, whether default rules are shared in a group, and similar questions. For the following examples, let us imagine that agents do their default reasoning individually. Agents do not communicate; however, they may (privately) follow the reasoning of other agents.

We will show two examples that will present what must be considered even in this simplified epistemic setting. The group of agents will contain just two agents, let us call them Alice (a) and Bob (b). Now, a (default) epistemic model is a structure $M = (W, R_a, R_b, V, X_a, X_b)$ and a normal default action used by the agent a, for example, is a structure $\mathbf{D}^a = (\mathbf{S}, \mathbf{R}_a, \mathbf{R}_b, \mathsf{pre}, \mathbf{X}_a, \mathbf{X}_b)$ where the behavior of this structure depends on the type of reasoning, see Example 2 for an additional commentary. If the reasoning of a does not depend on the activity of b, then a normal default action corresponding to a normal default $\frac{\varphi:\psi}{\psi}$ used by the agent a is the structure $\mathbf{D}^a = (\mathbf{S}, \mathbf{R}_a, \mathbf{R}_b, \mathsf{pre}, \mathbf{X}_a, \mathbf{X}_b)$ where

- $\mathbf{S} = \{\mathsf{s}, \mathsf{t}\}$
- $(\mathsf{s}, \mathsf{t}) \notin \mathbf{R}_a$, but $(\mathsf{s}, \mathsf{s}) \in \mathbf{R}_a$ and $(\mathsf{t}, \mathsf{t}) \in \mathbf{R}_a$
- $(\mathsf{x}, \mathsf{y}) \in \mathbf{R}_b$, for each $\mathsf{x} \in \mathbf{S}$ and $\mathsf{y} \in \mathbf{S}$
- $\mathsf{pre}(\mathsf{s}) = \psi$ and $\mathsf{pre}(\mathsf{t}) = \neg\psi$
- $\mathbf{X}_a = \{\mathsf{s}\}$
- $\mathbf{X}_b = \mathsf{S}$

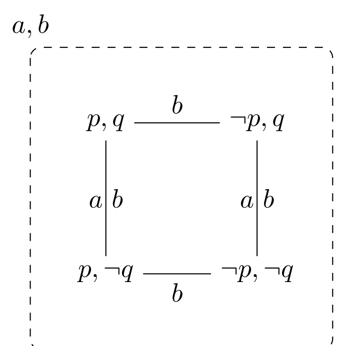
It means that b's accessibility relation and set of designated worlds will not be changed.

Example 1 In the first example we present the situation where Bob has a default theory $\left(\emptyset, \left\{\frac{:K_a p}{K_a p}\right\}\right)$, i.e., Bob has no hard information, but a default rule on Alice's knowledge of (an atomic fact) p. Consider the following epistemic model:

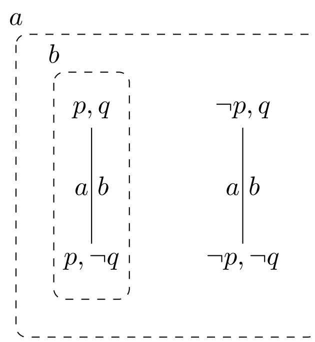


Neither Bob nor Alice know anything about p or q. All four epistemic worlds are indistinguishable for them and their sets of designated worlds are the same (indicated by the dashed line). The default action $(: K_a p / K_a p)^b$ is not applicable by Bob in this scenario. He does not admit the epistemic possibility that Alice knows p, see the second condition in Definition 7.

If Alice can distinguish p-worlds and $\neg p$-worlds, as in the following figure, then the action is applicable by Bob.

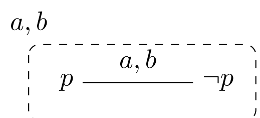


After we apply the default rule we obtain a new epistemic model:

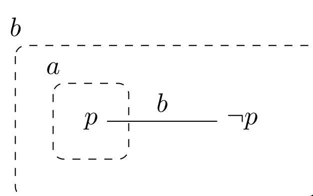


Bob's set of designated worlds has changed and from now on he works with the fact that Alice knows p.

Example 2 The second example presents private and semi-private reasoning and their differences. Recall our example with Anne, a student of logic at Faculty of Arts. Let's consider our two agents, Alice and Bob, who discuss whether Anne likes mathematics or not. They both know the hard information that Anne studies logic at Faculty of Arts. If we label the fact that Anne likes mathematics as p, the situation might look like this:

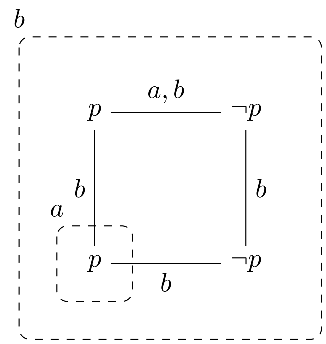


Alice and Bob consider both p and $\neg p$ possible and include both situations in their respective designated sets. After a short discussion Alice decides that she prefers to use the default reasoning that Anne indeed does like mathematics. Bob has access to the same default rules as Alice but has not decided yet.



In this model Alice's designated set contains only the state where p holds. Bob's designated set and accessibility relations were unchanged. However Alice's deduction was in some sense public, or semi-private. Bob knows that Alice knows something new, i.e., the formula $K_b(K_a p \vee K_a \neg p)$ holds in the whole model, resp. in the submodel generated by b's set of designated worlds. We may obtain this model simply by applying the default action $(: p/p)^a$.

Alternatively we may consider that Alice's reasoning is private. In this case Bob's knowledge will in some sense decrease.



Now, Bob has no clue about Alice's knowledge. He doesn't even know whether Alice 'used' a default reasoning or not.

This situation (private reasoning) may be achieved by a slight change in the definition of the default action model. Along the two action states $\{\mathsf{s}, \mathsf{t}\}$ there is a new state u whose precondition is $(p \vee \neg p)$. Bob's accessibility relation will be universal, i.e., any two states are in the relation $\mathbf{R}_b$.[22] The action model structure for a's private use of $(: p/p)^a$ is the following $\mathbf{D}^a = (\mathbf{S}, \mathbf{R}_a, \mathbf{R}_b, \mathsf{pre}, \mathbf{X}_a, \mathbf{X}_b)$ where

- $\mathbf{S} = \{\mathsf{s}, \mathsf{t}, \mathsf{u}\}$
- for all $\mathsf{x} \in \mathbf{S}$, just $(\mathsf{x}, \mathsf{x}) \in \mathbf{R}_a$
- $(\mathsf{x}, \mathsf{y}) \in \mathbf{R}_b$, for all $\mathsf{x} \in \mathbf{S}$ and $\mathsf{y} \in \mathbf{S}$
- $\mathsf{pre}(\mathsf{s}) = p$ and $\mathsf{pre}(\mathsf{t}) = \neg p$ and $\mathsf{pre}(\mathsf{u}) = (p \vee \neg p)$
- $\mathbf{X}_a = \{\mathsf{s}\}$
- $\mathbf{X}_b = \mathsf{S}$

4 Conclusion and further research

This paper focuses on showing that normal default reasoning and action models can share a common ground. We investigated the relationship between normal defaults and action models and proposed a way to 'translate' normal default rules into (default) actions in the framework of action models logic. The

[22] Compare the action model and the resulting epistemic model in [7, p. 153, Example 6.13].

presented system shows the role of normal defaults in a simple semi-private reasoning or, alternatively, in a completely private way of thought of an agent.

A lot of further research remains in this field. One may obviously investigate general default rules or semi-normal defaults. An interesting generalization stems from the behavior of group epistemic modalities like common knowledge and distributive knowledge that are important for communication among agents. This was mentioned in the previous section.

Another thing to consider is that some rules might be more informative or more 'correct' than other rules. We would want the agents to apply these better defaults before they apply any others. This can be achieved by a preference function. Each agent would have a preference function that would order each of her defaults by preference. This preference function might even be interactive. For example if agent a sees that agent b used default d_1, she might be more inclined to use the same default d_1 instead of d_2. These ideas bring us to a question whether there is a correspondence to belief revision and to a (technical) problem of the combination of default actions with other actions in this framework.

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