Logic of Questions
Part II
Inferential Erotetic Logic (IEL)
(Autumn School of Logic, Pec pod Snezkou, October 20-25, 2004, Michal Pelis)

Some *declarative sentences* (no free variables) are assigned to a question - *direct answers.*

(c1) Each question has at least two direct answers.

(c2) Each finite and at least two-element set of sentences is the set of direct answers to some question.

Language $\mathcal{L}_1$

$$\mathcal{L}_1 = \mathcal{L}_{PL1=} \cup \{?, \{, \}\}$$

(questions of the *first kind*)

$$Q = ?\underbrace{\{\alpha_1, \ldots, \alpha_n\}}_{dQ}$$

dQ is finite and $\forall i, j(i \neq j \Rightarrow \alpha_i \not\equiv \alpha_j)$

Language $\mathcal{L}_2$

$$\mathcal{L}_2 = \mathcal{L}_1 \cup \{S, O, U\}$$

(questions of the *second kind*)

$$Q = ?S(P(x))$$

for c (individual constant) is $P(x/c) \in dQ$

$$Q = ?O(P(x))$$

$$Q = ?U(P(x))$$

Semantics—Entailments

Γ, Δ are sets of flas and φ is a fla:

$$\Gamma \models \varphi$$

For each $\mathbf{M}$: if $\mathbf{M} \models \Gamma$, then $\mathbf{M} \models \varphi$.

$$\Gamma \mid\models \Delta$$

(multi-conclusion entailment) For each $\mathbf{M}$: if $\mathbf{M} \models \Gamma$, then at least one fla in Δ is valid in $\mathbf{M}$.

1. Question Q is *sound* relative to $\mathbf{M}$ iff $\exists \alpha \in dQ$ such that $\mathbf{M} \models \alpha$.
2. Question Q is *sound relative to* Γ iff $\Gamma \mid\models dQ$.
3. Question Q is *informative relative to* Γ iff $\forall \alpha \in dQ$ is $\Gamma \not\models \alpha$.
4. Γ *evokes* question Q iff 2 and 3.
5. Question Q is *safe* iff $\emptyset \mid\models dQ$, else it is *risky*.

Presupposition
A set of all presupposition of a question Q:

$$\text{Pres}Q = \{\varphi | \forall \alpha \in dQ (\alpha \models \varphi)\}.$$

Presupposition φ is a *prospective presupposition* iff $\varphi \mid\models dQ$.

- Each question has a presupposition.
- $\text{PPres}Q \subseteq \text{Pres}Q$
- If $\varphi, \psi \in \text{PPres}Q$, then $\varphi \equiv \psi$.

Classes of questions

- *Normal* questions: $\text{Pres}Q \mid\models dQ$.
- *Regular* questions: $\text{PPres}Q \neq \emptyset$.
- *Self-rhetorical* questions: $\text{Pres}Q \models \alpha$ for some $\alpha \in dQ$.
- *Proper* questions: normal and not self-rhetorical.

Language $\mathcal{L}_1$ and semantics

- Interpretations for $\mathcal{L}_1$ are the same as for $\mathcal{L}_{PL1=}$.
- $\text{TAUT}_{\mathcal{L}_1} = \text{TAUT}$
- Both entailments are compact.
- For each Q:
  - $\text{PPres}Q \neq \emptyset$,
  - Q is NORMAL and REGULAR.

Language $\mathcal{L}_2$ and semantics

Normal model: there is an individual constant (name) for each element of M.

- If $\Gamma \in$ FIN, then Γ has a model iff Γ has a normal model.
- Both entailments are *not* compact.
- For each question Q of the first kind and existential and open question of the second kind:
  - PPres$Q \neq \emptyset$,
  - Q is NORMAL and REGULAR.

Evocation

Γ **evokes** a question Q ($\Gamma \models Q$) iff

1. $\Gamma \mid\models dQ$,
2. $\forall \alpha \in dQ(\Gamma \not\models \alpha)$.

- If $\Gamma \models Q$, then Pres$Q \subseteq$ CnΓ.
- $Q \in$ SAFE:
  - Pres$Q =$ TAUT
  - Each Γ evokes Q if $dQ \cap$ Cn$\Gamma = \emptyset$.
- $Q \in$ NORMAL: $\Gamma \models Q$ iff Pres$Q \subseteq$ CnΓ and $\Gamma \not\models \alpha$ (for each $\alpha \in dQ$).
- $Q \in$ REGULAR: $\Gamma \models Q$ iff there is $\varphi \in$ PPres$Q$ such that $\Gamma \models \varphi$ and $\Gamma \not\models \alpha$ (for each $\alpha \in dQ$).
- Let entailment be compact: $\Gamma \models Q$ iff there is a finite subset Δ of dQ ($|\Delta| > 1$) such that $\Gamma \mid\models \Delta$ and $\Gamma \not\models \alpha$ (for each $\alpha \in dQ$).
- Let $\Gamma \models Q$ and $\Delta \subseteq \Gamma \subseteq \Sigma$, then
  - $\Delta \models Q$ if $\Delta \mid\models dQ$,
  - $\Sigma \models Q$ if $\Sigma \not\models \alpha$ for each $\alpha \in dQ$.

Partial answer

ψ is a *partial answer* to a question Q iff

1. $\forall \alpha \in dQ(\psi \not\equiv \alpha)$,

2. There is a non-empty proper subset of dQ ($\emptyset \neq \Delta \subset dQ$) and $\psi \mid\models \Delta$ and $\forall \alpha \in \Delta(\alpha \models \psi)$).

- If there is a partial answer ψ for Q and $\Gamma \models \psi$, then $\Gamma \models Q$.

Erotetic implication

$\Gamma, Q \models Q_1$ iff

1. $\forall \alpha \in dQ(\Gamma \cup \{\alpha\} \mid\models dQ_1)$,
2. $(\forall \beta \in dQ_1)(\exists \Delta \subset dQ)(\Delta \neq \emptyset$ and $\Gamma \cup \{\beta\} \mid\models \Delta)$.

Example:

Q: Where did K. leave for: Paris, or London?

Γ: If K. left for London, then he departed in the morning. If K. left for Paris, then he departed in the evening.

Q_1: When did K. depart: in the morning, or in the evening?

Reducibility of questions of the first kind to the simple yes-no questions

$$(\Gamma), ?\{\alpha_1, \ldots, \alpha_n\} \models ?\alpha_j$$

where $1 \leq j \leq n$ and $\bigvee_1^n \alpha_i \in \Gamma$ if question is not safe.

$$?\alpha_j \models ? \pm |\beta_1, \ldots, \beta_k|$$

where each β_i is a subformula of α_j.

$$? \pm |\beta_1, \ldots, \beta_k| \models ?\beta_i$$

where $1 \leq i \leq k$.

$$?\neg\beta \models ?\beta$$