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SET OF ANSWERS METHODOLOGY IN EROTETIC EPISTEMIC LOGIC

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ABSTRACT
This paper is an introduction to the methodology of questions' formalization based on sets of direct answers. Our approach corresponds to a widely accepted idea that the meaning as well as the inferential role of questions are associated with answerhood conditions. In addition, we show advantages of set-of-answers methodology for logic of questions in the framework of (dynamic) epistemic logic.
Keywords: epistemic logic, erotetic logic, questions.

1. Introduction

Questions and posing of questions are inherent parts of communication. Recently, on the one hand, some logical tools for an analysis of communication were developed. These tools are usually associated with knowledge changes of idealized agents. On the other hand, whenever we speak about the meaning of questions informally, we use epistemic terms too. In particular, a question expresses an agent's ignorance. The simultaneous development of logics of questions nowadays initiated a natural aspiration to combine logic of questions with epistemic logic. Following this viewpoint we are going to present the meaning of questions in the framework of any epistemic-like logic. That is the reason why we use the term *erotetic epistemic logic* in the title. This means that logic of questions is an extension of a background epistemic system.

Our aim is to present a formalization of questions that can be used in any epistemic logic. We do not want to provide a logical analysis of natural language questions. However, the formalization, called set-of-answers methodology, makes it possible to separate the epistemic meaning of a question from pragmatic aspects of a question.

In section 2 we discuss various methodological approaches to a question formalization in logic. Common aspects are emphasized, especially, a question-and-answers relationship and the role of answerhood conditions in inferences with questions. Next section introduces a formalization based on sets of direct answers. We justify this methodology by its use in a general epistemic setting in section 4.

2. Questions, answers, and inferences

Declarative sentences usually have their formal counterparts in logic and play an important role in argumentation. We often see logic to be primarily a study of inferences. Inferential structures are studied in formal systems, which can differ in the formalization of declaratives as well as in admitting or rejecting of some principles.

We believe that the dealing with questions in the logical framework will be fully justified if we show that questions can play an important and autonomous role in inferences. Perhaps this point may be considered as the most important to justify the existence of erotetic logic.[1]

This section provides a brief overview of methodological problems in (recent) approaches to erotetic logic and leads the reader to a justification of a methodology based on sets of answers.

2.1 Questions and answers

Let us imagine a group of three friends: Ann, Bill, and Catherine. Each of them has one card and nobody can see the cards of the others. One of the cards is the Joker and everybody knows this fact.[2] Then

> Who has the Joker?

is a reasonable sentence in this situation. We recognize it as an *interrogative sentence* because of its word order and the question mark. The hearing or uttering of an interrogative is followed by intonation and interrogative pronounce.

However, an interrogative sentence includes more: a pragmatic aspect. A question is a "request to an addressee to provide the speaker with certain information", this is *interrogative speech act* [3, p. 1057]. Pragmatically oriented approaches emphasize the roles of a speaker and an addressee. The roles seem, on the one hand, to be outside of the proper meaning of interrogatives. On the other hand, they are often understood as to be a crucial aspect in analyses of questions; this can be the reason why some logicians argue against some variants of logic of questions.

No matter what is our starting position if we want to work with interrogatives in the framework of a formal system, we are obliged to decide at least the following two problems:

(1) How to formalize questions?
(2) What is the (formal) semantics of questions?

Reviewing the history of erotetic logic, there is no unique solution. There are many approaches to the formalization of questions and every approach varies according to what is considered as important. Logic of questions is multiparadigmatic. David Harrah [4, pp. 25–26] illustrates it nicely by examples of so called 'meta-axioms'. He groups them into three sets according to the degree of an acceptance by erotetic logicians:

[1] In this paper we use the term *erotetic logic* in the same meaning as *logic of questions*. A discussion on both terms can be found in [5].

[2] In the epistemic setting we will expect more: the rules are *commonly known*.

(1) The first group includes meta-axioms accepted in almost all systems. Harrah calls them *absolute axioms*. For example:
   (a) Every question has at least one partial answer.
   (b) (In systems with negation) For every statement P, there exists a question Q whose direct answers include P and the negation of P.
   (c) Every question Q has a presupposition P such that: P is a statement, and if Q has any true direct answer, then P is true.
(2) The second group, *standard axioms*, is often accepted, but not in all systems.
   (a) Every question has at least one direct answer.
   (b) Every direct answer is a statement.
   (c) Every partial answer is implied by some direct answer.
   (d) Every question is expressed by at least one interrogative.
   (e) Each interrogative expresses exactly one question.
   (f) Given an interrogative I there is an effective method for determining the direct answers to the question expressed by I.
(3) The last group is called *eccentric axioms*. The following examples of such axioms are accepted only in some interrogative systems:
   (a) If two questions have the same direct answers, then the two questions are identical.
   (b) Every question Q has a presupposition that is true just in case some direct answer to Q is true.

Let us notice the terminology, the difference between *interrogative (sentence)* and *question* has been just introduced by standard axioms. The first term mostly refers to the type of a sentence and the second one is a bit more complex. A question is expressed by an interrogative (sentence) and can be 'posed', 'asked', etc., cf. [5]. Although we use *interrogative* and *question* in the same meaning here, the term *interrogative sentence* is reserved for a natural-language sentence, if necessary.

What seems to be common to all approaches is that questions are something structured and closely connected with their answers. We hardly find a propositional theory where questions are unstructured as atomic propositions are. The relationship

question — answer(s)

is a very conspicuous sign and the meaning of questions is always closely connected to *answerhood conditions*.

Since an answer to a question is often represented by a declarative, the natural starting point of many erotetic theories is a standard formal system for declaratives.

> "Any first-order language can be supplemented with a question-and-answer system" [15, p. 37].

This broadly accepted statement combines the possible solution of both the formal shape and the meaning of a question. Questions' autonomy depends on the chosen solution. Andrzej Wiśniewski distinguishes two basic groups of erotetic theories: *reductionist* and *non-reductionist*. Roughly speaking, non-reductionism is characterized by questions that “are not reducible to expressions of other syntactic categories” [15, p. 40], see subsection 3.1 too. The boundary between both groups is vague. Perhaps only pure pragmatically oriented approaches belong to the radical reductionism with a complete rejection of questions as a specific entity in formal logic.[3]

2.2 Inferences with questions

Although there are discussions whether it is necessary to work with questions as a new specific entity in a formal system, almost all theorists agree that questions play a specific role in inferences. Let us come back to our group of friends. The situation, where

either Ann has the Joker or Bill has the Joker or Catherine has the Joker,

can raise to the question

Q: *Who has the Joker?*

A question is raised (inferred) from a declarative or from a set of declaratives. What would make this raising reasonable? ‘Answerhood conditions’ is the answer. Any ‘reasonable’ answer to the question Q is connected to the declarative context.

Another kind of inferential structure could be based on declaratives as well as questions among premises. For example, from

Q: *Who has the Joker?*

and

Γ: *The only person from London has the Joker.*

can be inferred the question

Q_1: *Who is from London?*

The relationship of the inferred question Q_1 and the question Q is based on their answerhood conditions again. An answer to Q can provide an answer to Q_1 with respect to the context Γ. Moreover, in this example, Q can be inferred from Q_1 and Γ as well. This shows that the relationship is dependent on various kinds of answerhood conditions and contexts.

Let us have Q the same, but the context is

A person from London has the Joker.

If two persons are from London and we gain their names in an answer to Q_1, then we receive only a *partial answer* to Q.[4] If each of the friends (or nobody) is from London and an answer to Q_1 does not provide any help for the answering of Q, it has no sense to speak of an inferential relation between Q and Q_1 with respect to this context.

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3 An example of one of the radical approaches is commented in [9].

4 Informally, a *partial answer* does not completely answer a question, but it eliminates some of the possible (and complete) answers.

The role of answerhood conditions in inferences among questions is obvious in the following example: From any (complete) answer to Q we obtain a (complete) answer to the question

Has Ann the Joker?

as well as for the questions

Has Bill the Joker?

and

Has Catherine the Joker?

Answerhood conditions of the previous three questions are *entailed* in the answerhood conditions of the question Q. They can be inferred from the answerhood conditions of Q. The question *Has Ann the Joker?* is *entailed* by Q.

We have just presented inference-like structures with questions as dependent on answerhood conditions. Now, still faced with the problem how to formalize the relationship of questions and answers, we will introduce a convenient solution based on a liberal set-of-answers methodology.

3. Set-of-answers methodology

We are going to solve the problem of the formal shape of questions simultaneously with the problem of the questions' semantics. The formalization of a question will be based on a set of 'specific' answers. Moreover, we suppose to show that such approach can also reflect some semantic and pragmatic requirements.

3.1 Semantics of questions

Some theories do not admit that questions could have an independent meaning in logic. Questions are paraphrased by declarative sentences; in particular, the question *Who has the Joker?* may be then paraphrased by

I ask you who has the Joker.

Another way is the paraphrasing by epistemic-imperative sentences:

Bring it about that I know who has the Joker!

The propriety of both paraphrases as a complete meaning of a question is rather problematic.[5] One of the problems is that these approaches are forced to work with a questioner and an addressee already on the basic level of questions' meaning. Of course, we expect to utilize the importance of a questioner and an addressee, but it should be a task for a chosen background system (e.g., dynamic logic) not for the general semantics of questions. We understand pragmatic aspects as a higher level analysis.

5 See also [5].

Nuel Belnap formulated three methodological constraints on the meaning of questions, which he used for a classification and evaluation of erotetic theories:[6]

(1) **Independence** Interrogatives are entitled to a meaning of their own.
(2) **Equivalence** Interrogatives and their embedded forms are to be treated on a par.
(3) **Answerhood** The meaning of an interrogative resides in its answerhood conditions.

The most important is the first requirement, which is the main sign of non-reductionist theories. To accept *independence* requirement means that we are obliged to look for a specific semantics of questions. The *equivalence* requirement is closely related to a semantic entailment and is dependent on the chosen semantics. The *answerhood* requires that the meaning of questions is related to the meaning of answers. In addition, we can work with the idea that the semantics of answers forms a good background for the study of the meaning of interrogatives.

Approaches, where answers are crucial for the meaning of questions, are displayed in the acceptance of the first postulate from the following list suggested by Charles Hamblin:[7]

(1) Knowing what counts as an answer is equivalent to knowing the question.
(2) An answer to a question is a statement.
(3) The possible answers to a question are an exhaustive set of mutually exclusive possibilities.

Each postulate may be argued against and the detailed discussion is available in [3]. However, according to David Harrah, adopting the first one is "the giant step toward formalization often called *set-of-answers methodology*" [5, section 2]. Although there is not only one kind of set-of-answers methodology (SAM, for short) in the literature, we will not make any survey here. In the next subsection we introduce an easy idea of a question representation by a set of *direct answers*.

3.2 Sets of answers

Generally, without any context, the question *Who has the Joker?* can be answered by expressions of the following form:

Ann.
Ann has it.
Ann has the Joker.
Ann and Bill.
⋮
Batman has the Joker.
⋮
Your friends.
People at this table.

6 Belnap, N. D., 'Approaches to the semantics of questions in natural language. Part I', Pittsburgh, 1981. Cited from [2, p. 3–4].

7 Hamblin, C. L., 'Questions'. Australasian Journal of Philosophy, 36(3): 159-168, 1958. Cited from [5].

⋮
Nobody.
⋮
etc.

The question seems to be answered if a (complete) list of Joker's owners is given. We can assume that answers are propositions; thus, the first three items in the list have the same meaning in the answering of the question.

From the viewpoint of propositional logic and in accordance with the first two Hamblin's postulates, we can understand every question closely connected with a set of (propositional) formulas—answers.

Furthermore, we can receive some of the following responses to the same question:

Ann hasn't the Joker.

or

I don't know who has the Joker.

The first one can be considered to be a *partial answer*; it removes some answers as impossible, in particular, all answers with Ann having the Joker. (Can you imagine any context when the answer is complete?)

The second one appears to bear another kind of information; an addressee says to a questioner that she has the same problem and would ask the same question. (We will return to this topic in subsection 4.1.)

If we had decided to represent every question by a complete set of its answers, we would not always have a clear and useful formalization of questions. Let us return to our example of three friends with cards. Considering the context and the question *Who has the Joker?*, a questioner expects one of the following responses:

α: *Ann has the Joker.*
β: *Bill has the Joker.*
γ: *Catherine has the Joker.*

or a response that leads to one of the just mentioned.

In fact, the question

Who has the Joker?

with respect to the context

Either Ann has the Joker or Bill has the Joker or Catherine has the Joker.

might be reformulated to

Q': *Who has the Joker: Ann, Bill, or Catherine?*

The answers α, β, and γ are understood as 'core' answers that form the meaning of the question Q'. We use the term *direct answers* for them. The sentence

δ: *Neither Ann nor Bill have the Joker.*

is an answer, from which γ is inferred thanks to the context; δ is a *complete answer* to Q'. Complete answers are 'solutions' of a question and the set of direct answers is a subset of the set of complete ones.

Our SAM is inspired by the syntactic representation of questions in *inferential erotetic logic* founded by Andrzej Wiśniewski.[8] We want to be very liberal and this leads us to considering questions to be sets of formulas, which play the role of direct answers. A (general) declarative language $\mathcal{L}$ is extended only by curly brackets ($\{,\}$) and question mark (?). A question is the following structure

$$?\{\alpha_1, \alpha_2, \ldots\}$$

where $\alpha_1, \alpha_2, \ldots$ are formulas of the extended language.

No wonder that we have to impose some restrictions on direct answers to keep their exclusive position. Such restrictions are mostly a combination of syntactic and semantic requirements. From the syntactic viewpoint and being inspired by the previous examples, we require the following:

(1) Formulas $\alpha_1, \alpha_2, \ldots$ are syntactically distinct.
(2) A set of direct answers has at least two elements.

Both restrictions introduce questions as 'tasks' with at least two distinct 'solutions'. Syntactical distinctness is a first step to the idea that direct answers form the 'core' of questions' meaning. In semantics we will require non-equivalence above that.

The most typical questions with only two direct answers are *yes-no questions*. The question

Has Ann the Joker?

has, in fact, the following two direct answers:

Yes. (*Ann has the Joker.*)
No. (*Ann has not the Joker.*)

Such question will be identified with the form $?\{\alpha, \neg\alpha\}$ and shortened as $?\alpha$. A yes-no question is a variant of a *whether question*, where an answer is a choice from two possibilities. The role of negation is considered to be very important in SAM. Negation is always related to a background system and receiving $\neg\alpha$ can mean more than 'it is not the case that α'—it expresses something like 'strict denial of α'.[9]

3.3 SAM in the recent history of erotetic logic

The logic of questions has, maybe surprisingly, a long history. F. Cohen and R. Carnap seem to be the first authors attempting to formalize questions in a logical framework—their attempts date back to the 1920s [5, p. 3]. The first 'boom' of logical approaches to questions took place in the 1950s (Hamblin, Prior, Stahl) and continued in the 1960s (Åqvist, Harrah, Kubiński). The first comprehensive monograph on questions [1] brought

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[8] The best overview of questions' formalization in inferential erotetic logic is in the chapter 3 of the book [15]. See the article [16] as well.

[9] Compare it with an interpretation of negation and compatibility relation in substructural logics, cf. [8] for an epistemic example.

into life many important terms used in erotetic logic so far. The late 1970s gave birth to influential reductionist theories: Hintikka's epistemic-imperative approach and Tichý's approach based on his *transparent intensional logic* [14].

We are not going to present a complete survey of erotetic theories. The reader can find a comprehensive overview of the history of erotetic logic in [5]. Erotetic theories with the main influence in this field of study are described in [15, chapter 2] as well. Moreover, both papers provide a good introduction to the terminology used in logic of questions and cover enough the history of erotetic logic till the 1990s. The period from the 1950s till 1990s is mapped in [4]. Mainly linguistic viewpoint with the detailed discussion about the semantics of questions and pragmatic approaches can be found in [3].

Most influential modern logics of questions with the important role of erotetic inferences are

- *inferential erotetic logic* (IEL) of Andrzej Wiśniewski and
- intensional approach of Jeroen Groenendijk and Martin Stokhof.

Both theories appears fully developed in the 1990s and we consider them giving birth to several approaches some of which are still influential.

Inferential erotetic logic

Wiśniewski's IEL is a complex system dealing with various interrogative inferential structures. The influence of Nuel Belnap's and Tadeusz Kubiński's works is apparent. Primarily it is based on classical logic and a formalization of questions, which is very similar to the introduced SAM.

Inferential erotetic logic accepts only the first two Hamblin's postulates and tries to keep the maximum of the (classical) declarative logic and its consequence relation. On the syntactic level of a considered formalized language, a question is assigned to a set of sentences (direct answers). Direct answers are declarative formulas, each question has at least two direct answers, and each finite and at least two-element set of sentences is the set of direct answers to some question [16, p. 11]. The important difference from our liberal SAM is that direct answers are declaratives. In case of finite versions of questions $?\{\alpha_1, \ldots, \alpha_n\}$ direct answers are supposed to be semantically non-equivalent. The class of 'finite questions' corresponds to the class of *questions of the first kind* in IEL [15].

In the original version of IEL, questions are not identified with sets of direct answers: questions belong to an object-level language and are expressions of a strictly defined form, but the form is designed in such a way that, on the metalanguage level (and only here), the expression which occurs after the question mark designates the set of direct answers to the question. Questions are defined in such a way that sets of direct answers to them are explicitly specified. The general framework of IEL allows for other ways of formalizing questions.[10]

Any declarative logic can be defined by its consequence relation as a set of pairs $\langle \Gamma, \Delta \rangle$, where Γ and Δ are sets of (declarative) formulas and Δ is usually considered to be a singleton. Inferential erotetic logic makes one step more and adds new consequence relations mixing declaratives and interrogatives. Conclusion relations among questions and declaratives are defined on the metalanguage level where the role of multiple-conclusion

[10] Personal communication with Andrzej Wiśniewski.

entailment is important.[11] The most important consequences are *evocation*, *erotetic implication*, and *reducibility*:

- *Evocation* is a binary relation $\langle \Gamma, Q \rangle$ between a set of declaratives Γ and a question.[12]
- *Erotetic implication* is a ternary relation $\langle Q, \Gamma, Q_1 \rangle$ between an initial question Q and an implied question Q_1 with respect to a set of declaratives Γ.[13]
- *Reducibility* is a ternary relation $\langle Q, \Gamma, \Phi \rangle$ between an initial question Q and a set of questions Φ with respect to a set of declaratives Γ.

Motivations and natural-language examples of these consequence relations are introduced in the literature. We mentioned examples of evocation as well as erotetic implication in subsection 2.2. Let us recommend texts [15, 16] for both evocation and erotetic implication, and reducibility is studied in [7, 15, 17]. The advantage of IEL is its possible generalization for non-classical logics. In [10] we presented slightly modified version open for such step.

Intensional erotetic logic

Groenendijk's and Stokhof's approach might be called *intensional erotetic logic*. The meaning of a declarative sentence is given by truth conditions. For simplicity, let us imagine *logical space*, which is understood as a set of all 'possible' *states* (we often call them 'possible worlds', 'indexes' or 'situations'). Intension of a declarative is then a set of states where the declarative is true. Extension of a declarative is its truth value in a given state.

Combining intensional semantics with the full acceptance of Hamblin's postulates we obtain the meaning of a question as a *partitioning* of logical space. In accordance with the third postulate, answers to a question form an exhaustive set of mutually exclusive propositions. Partitioning of logical space is the intension of a question. Extension of a question in a given state is the answer, which is true there. This is very similar to SAM we introduced in IEL with added restrictions posed by all Hamblin's postulates.

Similarly to IEL, we can naturally define many important terms: partial answer, complete answer, informative value of answers, and entailment relation between questions. A question Q *entails* a question Q_1 iff each answer to Q implies an answer to Q_1. It means that the question Q provides a refinement of the partitioning given by the question Q_1. See [2] and [3].

Both IEL as well as intensional approach of Groenendijk and Stokhof influenced our idea of epistemic logic enriched by questions.

4. Epistemic aspects

It is very natural to see a question expressing an ignorance of a questioner delivered to an addressee. We mentioned it as the pragmatic aspect of questions. Looking for an

11 We say that Γ multiple-conclusion entails Δ ($\Gamma \Vdash \Delta$) iff each model of Γ models at least one formula in Δ.

12 It corresponds to the idea represented by the first example in subsection 2.2. Γ multiple-conclusion entails the set of direct answers to the question Q and, simultaneously, none of the direct answers is entailed by (the context) Γ.

13 See the second example in subsection 2.2.

epistemic counterpart of questions in the history of erotetic logic, the most known are *epistemic-imperative* approaches of Åqvist and Hintikka. These theories are reductionist ones, questions are translated into epistemic-imperative statements. Let us remind Hintikka's analysis [6] and the paraphrase *Bring it about that I know who has the Joker!* for the question *Who has the Joker?*. Both approaches are based on the idea of a questioner who does not know any answer to a question and who calls for a completion of knowledge.[14]

We said (subsection 3.1) that the pragmatic aspect of questions is not on the basic level of questions' meaning analysis. In this section we show set-of-answers methodology applied in a general epistemic framework. It opens the space for the formalization of communication with questions inside dynamic epistemic logics. Epistemic analysis of questions has then two important parts. The first one (individual) works with the knowledge and ignorance of a questioner. The second one (group) considers an exchange of information in a group of agents.

Let us return to the example with three friends holding cards. If, in particular, Catherine wants to find out where the Joker-card is, then she can ask

Who has the Joker: Ann, or Bill?

From our set-of-answers methodological viewpoint, the question has a two-element set of direct answers:

α: *Ann has the Joker.*
β: *Bill has the Joker.*

The question is 'reasonable' in this situation. Asking it Catherine expresses that she

(1) does not know what is the right answer to the question,
(2) considers the answers α and β to be possible, and, moreover,
(3) presupposes what is implicitly included in the set of direct answers $\{\alpha, \beta\}$, i.e., it must be the case that just either Ann has the Joker or Bill has it.

Just introduced SAM makes it possible to specify expectations and presuppositions of a questioner. A questioner expresses not only the ignorance of Joker's holder, but the presupposition that the holder must be either Ann or Bill. The set of direct answers informs what answers are considered as possible and, on top of that, what is the questioner's rank of all complete answers.

4.1 Communication with questions

To employ the epistemic aspects of a question we have to indicate a questioner and an addressee. In accordance with the thesis that epistemic aspects are up to the background system, we enrich the language of any multi-agent propositional epistemic logic by curly brackets and indexed question mark ($?_x$) where x is the 'name' of an agent or of a group of agents; x indicates the questioner(s). In particular, Catherine's question can be formalized by

$$?_c\{\alpha, \beta\}$$

14 More details are in [5] and [15, chapter 2].

The role of an addressee is then a task for dynamic versions of epistemic logic. For example, in the framework of public announcement logic, see [12, 13], Catherine asks the question publicly among her friends and the addressee is the whole group of friends-agents $\{\text{Ann, Bill, Catherine}\}$.

Asking (publicly) the question $?_c\{\alpha, \beta\}$ the addressee can obtain the following information:

(1) The agent c does not know whether α or β.
(2) The agent c considers α and β as her epistemic possibilities.
(3) The agent c expects a complete answer leading either to α or to β.

If c is not a liar, these items can be understood as delivering a good picture of c's epistemic state to an addressee. It is important for a cooperative communication that an agent fully reveals her ignorance as well as expectations. A publicly asked question can then solve a problem (without being answered). Let us suppose, Ann has the Joker and Catherine has just asked the question $?_c\{\alpha, \beta\}$. Then Bill can infer:

> Catherine doesn't know who has got the Joker and she expects that either I have got it or Ann has got it. I haven't got the Joker. Therefore Ann must have it.

Catherine's publicly asked question was *informative* for Bill. He had the same problem *Who has the Joker?*, which is now solved.[15]

I don't know answer A question is often understood as a 'problem to solve'. Whenever we ask a question Q and someone answers *I don't know*, we can interpret it: she has 'the same' problem, she would ask 'the same' question. In fact, we received an answer to a question

> *Would you ask the question Q?*

Our liberal set-of-answers methodology makes it possible to deal with such questions; questions can be among direct answers as well.

Let us return to our group of friends. Whenever Bill wants to find out whether *Who has the Joker?* is a task for Catherine, he could ask

$$?_b\{?_c\{\alpha, \beta\}, \neg ?_c\{\alpha, \beta\}\}$$

It is a yes-no question where the first direct answer means that Catherine *does not know* who has the Joker or, rather, that she would ask the question formalized by $?_c\{\alpha, \beta\}$. The second one means that Catherine would not ask the question $?_c\{\alpha, \beta\}$. The reason can be that she knows a complete or a partial answer or she does not expect an answer just from the set $\{\alpha, \beta\}$.

As we saw in the previous paragraph, Bill can use such 'question about question' to gain an information without revealing his own ignorance structure about the Joker-card holder. The context, where each agent knows the card he or she holds, causes that the interrogative sentence *Who has the Joker?* can have a specific form for each agent in SAM; in particular, $?_c\{\alpha, \beta\}$ for Catherine and $?_b\{\alpha, \gamma\}$ for Bill. These questions would not correspond to a situation when nobody of them knows his or her own card. Then the question *Who has the Joker?*, in the SAM formalization $?_G\{\alpha, \beta, \gamma\}$, is a problem for each

[15] The formal definition of informative questions in the framework of public announcement logic can be found in [11, 12, 13].

member of the group of agents $G = \{\text{Ann, Bill, Catherine}\}$. The formalization of questions in SAM can help to deliver the ignorance as well as knowledge structure of an agent (group of agents) in a given context to addressees.

4.2 Conclusion remarks

We presented an easy formalization of questions which seems to be useful for the analysis of communication in the framework of various dynamic epistemic logic. First, introduced SAM provides an epistemic meaning of questions dependently on the background epistemic system. And, second, any dynamic extension of the background epistemic logic displays the role of pragmatic aspects of interrogatives.

A question becomes a component of epistemic logic with the meaning related to Kripke structures, cf. [11, chapter 3] and [12]. In multi-agent versions *common* and *distributive* knowledge may be employed. Common knowledge is important in models of a shared context in a group of agents. Distributive (also *implicit*) knowledge is useful in modeling of a hidden information, which can be obtained by a communication among agents. We started the work in this field and in the paper [13] we investigated the behavior of questions in the framework of *public announcement logic*.

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References

[1] Nuel Belnap and Thomas Steel. *The Logic of Questions and Answers*. Yale, 1976.

[2] Jeroen Groenendijk and Martin Stokhof. Partitioning logical space. Annotated handout, ILLC, Department of Philosophy, Universiteit van Amsterdam, 1990. Second European Summerschool on Logic, Language and Information; Leuven, August 1990; version 1.02.

[3] Jeroen Groenendijk and Martin Stokhof. Questions. In J. van Benthem and A. ter Meulen, editors, *Handbook of Logic and Language*, pages 1055–1125. Elsevier, Amsterdam, 1997.

[4] David Harrah. On the history of erotetic logic. In Andrzej Wiśniewski and Jan Zygmunt, editors, *Erotetic Logic, Deontic Logic, and Other Logical Matters*, volume 17, pages 19–27. Wydawnictwo Uniwersytetu Wrocławskiego, 1997.

[5] David Harrah. The logic of questions. In D. Gabbay and F. Guenthner, editors, *Handbook of Philosophical Logic*, volume 8, pages 1–60. Kluwer, 2002.

[6] Jaakko Hintikka. *The Semantics of Questions and the Questions of Semantics*. North-Holland Publishing Company, 1976.

[7] Piotr Leśniewski and Andrzej Wiśniewski. Reducibility of questions to sets of questions: Some feasibility results. *Logique & Analyse*, 173–175:93–111, 2001.

[8] Ondrej Majer and Michal Peliš. Epistemic logic with relevant agents. In Michal Peliš, editor, *The Logica Yearbook 2008*, pages 123–135. College Publications, 2009.

[9] Michal Peliš. Questions and logical analysis of natural language: the case of transparent intensional logic. *Logique & Analyse*, 185–188:217–226, 2004.
[10] Michal Peliš. Consequence relations in inferential erotetic logic. In Marta Bílková, editor, *Consequence, Inference, Structure*, pages 53–88. Faculty of Arts, Charles University, 2008.
[11] Michal Peliš. *Logic of Questions*. PhD thesis, Faculty of Arts, Charles University in Prague, 2011.
[12] Michal Peliš and Ondrej Majer. Logic of questions from the viewpoint of dynamic epistemic logic. In Michal Peliš, editor, *The Logica Yearbook 2009*, pages 157–172. College Publications, 2010.
[13] Michal Peliš and Ondrej Majer. Logic of questions and public announcements. In N. Bezhanishvili, S. Löbner, K. Schwabe, and L. Spada, editors, *Eighth International Tbilisi Symposium on Logic, Language and Computation 2009*, LNCS, pages 145–157. Springer, 2011.
[14] Pavel Tichý. Questions, answers, and logic. *American Philosophical Quarterly*, 15:275–284, 1978.
[15] Andrzej Wiśniewski. *The Posing of Questions: Logical Foundations of Erotetic Inferences*. Kluwer, 1995.
[16] Andrzej Wiśniewski. Questions and inferences. *Logique & Analyse*, 173–175:5–43, 2001.
[17] Andrzej Wiśniewski. Reducibility of safe questions to sets of atomic yes-no questions. In J.J. Jadecki and J. Paśniczek, editors, *Lvov-Warsaw School: The New Generation*, pages 215–236. Rodopi, Amsterdam/NY, 2006.