Erotetic Epistemic Logic

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Abstract

This paper presents a logic of questions developed as an extension of (S5) epistemic logic. We discuss many features that are important for erotetic logic (formalization and semantics of questions, answerhood conditions, and inferential structures with questions). The aim is to introduce an erotetic system which corresponds well with epistemic terms and can form an appropriate background for dynamic approaches in epistemic logic.

Keywords: epistemic logic, erotetic implication, erotetic logic, logic of questions

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1 Introduction

One of the basic ways to complete someone's knowledge is the posing of questions. Although questions differ from declaratives, they also play an important role in reasoning. They appear in inferences (among premises as well as conclusions). In the recent history of erotetic logic we have seen success in finding a good position of formal approaches to questions and in the study of inferences based on them.[1] Such approaches made it possible to incorporate questions into formal systems and not to lose their specific position in inferences. This brings us to an important point. On the one hand, questions are specific entities and they bear some special properties. On the other hand, they are used in formal systems that focus on reasoning.

Our main aim is to present a possible erotetic enrichment of epistemic logic. In this paper we address issues previously examined in the third chapter of [7] where we presented some ideas how to work with questions in the framework of epistemic logic. In the book [7], our aim was to prepare a background for dynamic epistemic logic with questions and group modalities.[2] Some recent discussions about this work brought us back to basic notions, concepts, and ideas of erotetic logic built in the framework of epistemic logic.

The approach we are going to present in this paper works with epistemic logic as a background. We omit the dynamic aspect of communication here. Main erotetic issues that will be analysed in this epistemic framework are:

- formalization of questions (as an extension of modal language),
- extension of satisfaction relation for questions,
- answerhood conditions,
- inferential relations with questions.

A specialist in erotetic logic will recognize the significant influence of Inferential Erotetic Logic (IEL) [13, 14] and the intensional approach from [4, 5]. Both approaches inspired questions' formalization and semantics of questions. IEL inspired also various inferential structures with questions. More details can be found in [7] where IEL is also introduced.

2 Epistemic logic with questions

Epistemic logic and its semantics are used for the modeling of knowledge and ignorance of an agent.[3] Logic in use will be the normal multi-modal logic $\mathsf{S5}$ with its standard relational semantics (Kripke frames and models).[4]

The language of classical propositional logic $\mathcal{L}_{cpl}$ is extended by the modalities $[i]$ and $\langle i \rangle$. The former modality (*epistemic necessity*) can be interpreted

[1] See, especially, [15], [5], [11], and [2, 1].
[2] Cf. chapter 4 in [7] and papers [8, 9].
[3] The term *agent* supplies a subject like human being, machine, etc.
[4] See, e.g., [3].

as ‘agent i knows that ...’. The latter one is an *epistemic possibility*, it can be read as ‘agent i considers ... possible’. We obtain a language $\mathcal{L}^K_{cpl}$ with a subset of signs for atomic formulas $\mathcal{P} = \{p, q, \dots\}$ and formulas defined in BNF as follows:

$$\varphi ::= p \mid \neg\psi \mid \psi_1 \wedge \psi_2 \mid [i]\psi \mid \langle i\rangle\psi$$

In multi-agent variants of epistemic logic we suppose that there is a finite set of agents $\mathcal{A}$.

Semantics is based on Kripke-style models for normal modal logics. A *Kripke frame* is a relational structure $\mathcal{F} = \langle S, R_i\rangle$ with a set of states (points, indices, possible worlds) S, and an accessibility relation $R_i \subseteq S^2$ for (every) $i \in \mathcal{A}$. The accessibility relations are equivalences. A *Kripke model* $\mathbf{M}$ is a pair $\langle \mathcal{F}, v\rangle$ where v is a valuation of atomic formulas. The satisfaction relation $\models$ is defined in the standard way:

1. $(\mathbf{M}, s) \models p$ iff $s \in v(p)$
2. $(\mathbf{M}, s) \models \neg\varphi$ iff $(\mathbf{M}, s) \not\models \varphi$
3. $(\mathbf{M}, s) \models \psi_1 \wedge \psi_2$ iff $(\mathbf{M}, s) \models \psi_1$ and $(\mathbf{M}, s) \models \psi_2$
4. $(\mathbf{M}, s) \models [i]\varphi$ iff $(\mathbf{M}, t) \models \varphi$, for each t such that sR_it

Modality $\langle i\rangle$ is understood to be dual to $[i]$: $\langle i\rangle\varphi \equiv \neg[i]\neg\varphi$. We use standard semantic terms: a formula φ is *valid in a model* $\mathbf{M}$ ($\mathbf{M} \models \varphi$) iff $(\mathbf{M}, s) \models \varphi$, for every $(\mathbf{M}, s)$; a formula φ is *valid* (*theorem*) iff $\mathbf{M} \models \varphi$, for every $\mathbf{M}$; and a formula φ is *entailed* by a set of formulas Γ ($\Gamma \models \varphi$) iff $\mathbf{M} \models \varphi$, for every model $\mathbf{M}$ where each $\gamma \in \Gamma$ is valid. We write $(\mathbf{M}, s) \models \Gamma$ whenever $(\mathbf{M}, s) \models \gamma$ for each $\gamma \in \Gamma$.

We will use the following abbreviations in the language:

- $\psi_1 \vee \psi_2$ for $\neg(\neg\psi_1 \wedge \neg\psi_2)$
- $\psi_1 \rightarrow \psi_2$ for $\neg(\psi_1 \wedge \neg\psi_2)$
- $\psi_1 \leftrightarrow \psi_2$ for $(\psi_1 \rightarrow \psi_2) \wedge (\psi_2 \rightarrow \psi_1)$

Now, let us introduce questions in this framework. In the next section, we introduce the term *askability* of a question by an agent in a state, which extends the definition of satisfaction relation to interrogative formulas.

2.1 Incorporating questions

For a motivation, let us imagine a group of three agents: Anne, Bond, and Carl. Each of them has one card and nobody can see the cards of the others. One of the cards is the Joker and everybody knows this fact.[5] Joker means that an agent was chosen for a mission. If Carl wants to find the Joker-card holder, it is reasonable to ask

[5] In the full epistemic setting we would expect more—it is *common knowledge* for this group of agents.

Who has the Joker?

Whenever we know the context of this example, we can interpret Carl's question as the following one:

Who has the Joker: Anne, or Bond?

Carl expects an answer that leads to one from the set:

$$\{\text{Anne has the Joker}, \text{Bond has the Joker}\}$$

These answers are understood as 'core' answers for the meaning of the question *Who has the Joker: Anne, or Bond?* and we will call them *direct answers*. Direct answers form a subset of *complete answers*. Complete answers are 'solutions' to a question and direct answers can be inferred from them. In our example, *Anne does not have the Joker* is a complete answer, which leads to the direct answer *Bond has the Joker*. Detailed discussion on the applied *set-of-answers methodology* can be found in [6].

We extend the epistemic language $\mathcal{L}_{cpl}^{K}$ with brackets $\{,\}$ and the question mark $?_x$, where x indicates the questioner(s), it can be a 'name' of an agent or a group of agents. The obtained language is labelled $\mathcal{L}_{cpl}^{KQ}$. Metavariables Q^x, Q_1^x, and similar will be used for interrogative formulas. If it is clear from the context, we will not use the index for the name of an agent (or group of agents).

A question Q^x is any formula of the form

$$?_x\{\psi_1, \ldots, \psi_n\},$$

where $dQ^x = \{\psi_1, \ldots, \psi_n\}$ is the set of direct answers to a question Q^x. Direct answers are formulas of our extended epistemic language $\mathcal{L}_{cpl}^{KQ}$ and questions can be among direct answers as well. We suppose that dQ^x is finite with at least two syntactically distinct elements.

There is neither a special philosophy nor an algorithm for building sets of direct answers. Next to the mentioned syntactic restrictions, semantics will also impose some other restrictions. Nonetheless, we are very liberal as for what can occur as a direct answer. A question Q^x is syntactically determined by dQ^x. Intuitively, a question $Q^i = ?_i\{\psi_1, \ldots, \psi_n\}$ is agent i's question:

> Is it the case that ψ_1 or is it the case that ψ_2 ... or is it the case that ψ_n?

In our arrangement, Carl's question can be formalized as $?_c\{p, q\}$, where atomic formula p formalizes *Anne has the Joker* and q formalizes *Bond has the Joker*. Asking publicly this question, Carl expresses that he:

1. *does not know* any direct answer to the question, nonetheless,
2. considers the listed answers to be *possible*, and, moreover,
3. *presupposes* that (either) p or q.

An agent-questioner provides the information of his or her ignorance (item 1) as well as the expected way to complete his/her knowledge (items 2 and 3). Item 2 says that there are two possibilities, and item 3 that one of the possibilities is expected. A question provides (publicly) information about the epistemic state of the agent who asks the question.[6]

The context where each agent knows what card he or she holds, creates a situation where the interrogative sentence *Who has the Joker?* can have a specific form for each agent; in particular, $?_c\{p, q\}$ for Carl and $?_b\{p, r\}$ for Bond where r formalizes *Carl has the Joker*. These questions would not correspond to a situation where not one of them knows his or her own card. For example, cards are on the table and nobody looked at them. Then the question *Who has the Joker?*, in the formalization $?_G\{p, q, r\}$, is a problem for each member of the group G. The formalization of questions can help to reveal (to an addressee) the ignorance as well as knowledge structure of an agent or a group of agents in a given context.

In the motivational example, we write that *either Anne or Bond has the Joker*, i.e., it must be the case that Anne has the Joker (and Bond not), or it must be the case that Bond has the Joker (and Anne not). This is also indicated by the comma in the interrogative sentence. However, in this example, it is a combination of Carl's presupposition and the influence of the context:

Just one Joker is distributed among the agents.

Almost the same question (from the formal point of view)

What is Peter: a lawyer or an economist?

bears a presupposition that Peter is at least one of the two possibilities (maybe, both of them) if there is no supplementary context. The role of context will be studied in details later on. Now, let us introduce the concept of satisfiability of a question in a particular state of a particular Kripke model.

It is clear that it makes little sense to speak about the truth or falsity of a question. We introduce instead a notion of *askability* of a question. Askability is based on the three conditions we have just mentioned informally.

Definition 1 (Individual askability)**.** *We say that* $Q^i = ?_i\{\alpha_1, \ldots, \alpha_n\}$ *is* ask-able *by an agent* i *in a state* $(\mathbf{M}, s)$ *and write*

$$(\mathbf{M}, s) \models Q^i$$

iff

1. $(\mathbf{M}, s) \not\models [i]\alpha$, *for each* $\alpha \in dQ^i$ *(non-triviality)*
2. $(\mathbf{M}, s) \models \langle i \rangle\alpha$, *for each* $\alpha \in dQ^i$ *(admissibility)*

[6] The fact that a listener can form a (partial) picture of the questioner's knowledge structure is utilized in communication analysis in [7, 8, 9]. There we studied communication based on public announcement logic.

3. $(\mathbf{M}, s) \models [i] \left(\bigvee_{\alpha \in dQ^i} \alpha \right)$ *(context)*

We can speak about *group askability* whenever the question is askable for each agent from a (non-empty) group $G \subseteq \mathcal{A}$.

Definition 2 (Group askability)**.** *We say that* Q^G *is askable by a group of agents* G *in* $(\mathbf{M}, s)$ *and we write* $(\mathbf{M}, s) \models Q^G$ *iff* $(\forall i \in G)((\mathbf{M}, s) \models Q^i)$.

It is worth to note here that the concept of askability describes epistemic conditions for a question in an epistemic state. It does not mean that an askable question is publicly asked.

Originally, the definition of askability was designed for the broader set of formal systems. Especially, for systems where a difference between non-satisfiability ($\not\models$) and satisfiability of negated formulas can occur. In the introduced background epistemic logic $\mathsf{S5}$, the first condition of Definition 1 can be rewritten as: $(\mathbf{M}, s) \models \langle i \rangle \neg \alpha$, for each $\alpha \in dQ^i$. We can regard the questioner as admitting the possibility of $\neg\alpha$ for each direct answer α to a question Q^i. Simultaneously, questions are introduced as having finite sets of direct answers. Our questions are, in fact, complex modal formulas, in particular, $?_i\{\alpha_1, \ldots, \alpha_n\}$ is equivalent to

$$(\langle i \rangle \neg \alpha_1 \wedge \ldots \wedge \langle i \rangle \neg \alpha_n) \wedge (\langle i \rangle \alpha_1 \wedge \ldots \langle i \rangle \alpha_n) \wedge [i](\alpha_1 \vee \ldots \vee \alpha_n) \qquad (1)$$

The language $\mathcal{L}^{KQ}_{cpl}$ has the same expressive power as the epistemic language $\mathcal{L}^{K}_{cpl}$. Questions are not independent objects here. They are semantically reducible to complex modal formulas, in the sense of having the same satisfiability conditions as these formulas. Nonetheless, it is caused by the choice of background logic $\mathsf{S5}$ and by finite sets of direct answers. We will discuss the requirement of finiteness in subsection 2.7.

Askable questions (by at least one agent, in a state) include neither contradictions nor tautologies among their direct answers. The former is excluded by the second condition of Definition 1 (admissibility) and the latter by the first one (non-triviality). No question is a theorem of the system.

Our agents are 'erotetically introspective'. They know whenever a question is askable by them.

Proposition 1. $(\mathbf{M}, s) \models Q^i$ *iff* $(\mathbf{M}, s) \models [i]Q^i$.

Proof. The implication from right to left is obvious. For the other one, we prove: If $(\mathbf{M}, s) \models Q^i$, then $(\forall t \in sR_i)((\mathbf{M}, t) \models Q^i)$ where $sR_i = \{s' : sR_i s'\}$ is an equivalence class. We know that $(\forall t \in sR_i)(sR_i = tR_i)$ in $\mathsf{S5}$. For contradiction, suppose $(\mathbf{M}, s) \models Q^i$ and $(\exists t \in sR_i)((\mathbf{M}, t) \not\models Q^i)$. If $(\mathbf{M}, t) \not\models Q^i$, then there are three options:

1. $(\exists \alpha \in dQ^i)((\mathbf{M}, t) \models [i]\alpha)$ or
2. $(\exists \alpha \in dQ^i)((\mathbf{M}, t) \models [i]\neg\alpha)$ or
3. $(\mathbf{M}, t) \not\models [i] \left(\bigvee_{\alpha \in dQ^i} \alpha \right)$

The first (second) option requires α ($\neg\alpha$) to be true in every state from tR_i, but $tR_i = sR_i$; it is in contradiction with the askability of Q^i in $(\mathbf{M}, s)$. The third option also leads to contradiction, it requires that there is a state from $tR_i = sR_i$ where the context condition $[i]\left(\bigvee_{\alpha\in dQ^i}\alpha\right)$ is not true. □

The formula $(Q^i \leftrightarrow [i]Q^i)$ is valid in our system. We see that if there is a question Q askable by i in a state $(\mathbf{M}, s)$, then this question is askable in each state of the afterset sR_i. Askable questions are askable in the whole indistinguishable surrounding of an agent. As a corollary we obtain

Fact 1. *The formula* $(\langle i\rangle Q^i \leftrightarrow Q^i)$ *is valid.*

A non-askable question (in a state) is not askable in the whole afterset.

2.2 Context condition

Let us return to the following question (posed by i and without any supplementary information):

What is Peter: a lawyer or an economist?

This question can be formalized by the formula $?_i\{l, e\}$, where l represents *Peter is a lawyer* and e represents *Peter is an economist.* Its askability in a state $(\mathbf{M}, s)$ requires (at least) a minimal substructure on the equivalence class sR_i consisting of (at least) two accessible states, one of them satisfies l (and does not satisfy e) and the other one satisfies e (and does not satisfy l). All states in the afterset sR_i must satisfy the presupposition $(l \vee e)$, because of the context condition. Of course, this is a minimal requirement given by askability conditions. The full afterset structure can contain another states, some of them may satisfy both l and e (Peter can be both a lawyer and an economist), but none of them satisfies $\neg l$ and $\neg e$; the question $?_i\{l, e\}$ does not allow *neither l nor e* as a possible answer (because of the context condition). Such an answer occurs, for instance, in the case of a question formalized by $?_i\{l, e, (\neg l \wedge \neg e)\}$ (we admit that Peter is neither a lawyer nor an economist). The context condition can be understood as a 'minimal' knowledge of an agent (in an epistemic state), which is provided by the formalization of a question.

An 'outside' context can have a restrictive influence on possible answers, as it is seen in the case of the question *Who has the Joker: Anne, or Bond?* where the additional context is: *Just one Joker is distributed among the agents.*

For some classes of questions, the context condition is not important.

Definition 3. *A question* Q^i *is* safe *iff* $\left(\bigvee_{\alpha\in dQ^i}\alpha\right)$ *is valid.*[7]

In the introduced formal system, *yes-no questions* are one of the examples of safe questions. They have the form $?_i\{\alpha, \neg\alpha\}$ and are written concisely as $?_i\alpha$.

[7] Questions that are not safe, will be called *risky.* The original concept of safe and risky questions comes from Nuel Belnap.

The askability of such a question in a state $(\mathbf{M}, s)$ is equivalent to the truth of the formula $(\langle i\rangle\alpha \wedge \langle i\rangle\neg\alpha)$ in $(\mathbf{M}, s)$. The same can be said about $?_i\neg\alpha$, both $?_i\alpha$ and $?_i\neg\alpha$ are equivalent. Yes-no questions can be seen as a 'contingency modality'.

Conjunctive questions are another example of safe questions. The shortest ones fall under the schema:

$$?_i\{(\alpha \wedge \beta), (\neg\alpha \wedge \beta), (\alpha \wedge \neg\beta), (\neg\alpha \wedge \neg\beta)\}$$

We write them as $?_i|\alpha, \beta|$.[8] These questions express the full ignorance of α and β. Similarly to yes-no questions, they have an exhaustive set of direct answers, and direct answers are *mutually exclusive*.

The term *safe question* speaks about safeness in a global sense. The context condition of a safe question is valid, i.e., true in every epistemic state s for every model $\mathbf{M}$. The 'local' counterpart of this term is *safeness in a state*, which corresponds to the truth of the context condition in a particular state:

Definition 4. *A question* Q^i *is* safe (for an agent i) in a state $(\mathbf{M}, s)$ *iff* $(\mathbf{M}, s) \models [i]\left(\bigvee_{\alpha \in dQ^i} \alpha\right)$.

Clearly, a question, which is askable in a state (for an agent), is safe in this state (for this agent).

In paper [12] we resigned ourselves to the context condition. Later on, see subsections 2.7 and 3.2, this idea will be supported by some problems we have with the context condition. In 3.2 we will see that any question can be transformed into a set of safe questions where the context condition is not important. Nonetheless, then we can loose some information that was borne by the initial question. The context condition is tightly connected with an agent's epistemic state (in a model). We will discuss this topic again in section 4.

2.3 Answerhood conditions

To break the *non-triviality* condition (in Definition 1) means that there is a direct answer which is 'known' to an agent (in a state of a model). In fact, an agent knows a direct answer whenever he or she knows a formula which entails some direct answer. Such a formula is called a *complete answer*. In the following definition we describe a situation when an agent knows complete (resp., direct) answer to a question Q in an epistemic state $(\mathbf{M}, s)$.

Definition 5. *A question* $Q^i = ?_i\{\alpha_1, \ldots, \alpha_n\}$ *is* answered *in* $(\mathbf{M}, s)$ *(for an agent i) iff* $(\mathbf{M}, s) \models \bigvee_{\alpha_j \in dQ^i} ([i]\alpha_j)$.

The case of unsatisfied *admissibility* condition is a bit different. Let us imagine that an agent knows α in a state s (of a model $\mathbf{M}$). Then, even if she does not know an answer to a question $?_i|\alpha, \beta|$ in s, it is not right to ask this question. Not all possibilities required by the admissibility condition are

[8] The short version of formalization is inspired by IEL.

available, in particular, states with $\{\neg\alpha, \neg\beta\}$ and $\{\neg\alpha, \beta\}$ are missing. Some answers to the question $?_i|\alpha, \beta|$ provide information that is superfluous to the state of the agent's knowledge. Formula α is not a (complete) answer to the question $?_i|\alpha, \beta|$, it excludes some of the possible direct answers. In [7], we call such formulas *partial answers.*

Definition 6. *A question* $Q^i = ?_i\{\alpha_1, \dots, \alpha_n\}$ *is* partially answered *in* $(\mathbf{M}, s)$ *(for an agent i) iff* $(\mathbf{M}, s) \models \bigvee_{\alpha_j \in dQ^i} ([i]\neg\alpha_j)$*.*

Similarly to Definition 5, we want to say that an agent knows a formula which entails some negated direct answers.

Answerhood conditions can bring non-compliance with one's intuition.[9] The concept of partial answerhood based on Definition 6 is the example. A partially answered question need not be 'solved', as it is required from the role of complete answers, and a questioner could want to ask this question until he or she receives a complete answer. However, we would like to employ questions in the conception of dynamic epistemic approach where a questioner that receives a partial answer to a question is obliged to pose a (slightly) changed question, which is then askable and reflects the obtained partial answer and updated knowledge.

In subsection 3.1 we will discuss another problem in the relationship of complete and partial answers.

2.4 Questions among direct answers

Our set-of-answers methodology accepts questions among direct answers. It can have a good (dynamic) epistemic meaning. Whenever we ask a question Q and someone answers

I don't know,

we can interpret this answer as he or she has 'the same' problem, he or she would ask 'the same' question. In fact, we received an answer to a question

Is the question Q askable for you?

In our introductory example, whenever Bond wants to find out whether *Who has the Joker?* is a task for Carl, he could ask

$$?_b\{?_c\{p, q\}, \neg?_c\{p, q\}\}$$

It is a yes-no question where the first direct answer means that Carl *does not know* who has the Joker because the question formalized by $?_c\{p, q\}$ is askable (by Carl, in the actual epistemic state). The second one means that Carl would not ask the question or, rather, the question $?_c\{p, q\}$ is not askable by him. The reason can be that he knows a complete or a partial answer or he is not in compliance with the presupposition $(p \vee q)$.

[9] Personal communication with Andrzej Wiśniewski.

As we have seen, Bond can use such 'question about question' to obtain information without revealing his own ignorance about the Joker-card holder. If Bond has not the Joker and obtains the true answer to his question on Carl's ignorance, it will be 'informative' for him.[10]

In the previous example, one agent (b) posed a question about askability of a question Q for another agent (c). Does it make sense that one agent asks himself or herself whether Q is askable for him or her?[11] Is the question $?_i Q^i$ askable by i in any state? The question $?_i Q^i$ is a yes-no question $?_i\{Q^i, \neg Q^i\}$ and it is equivalent to the modal formula $(\langle i\rangle Q^i \wedge \langle i\rangle \neg Q^i)$.[12]

Fact 2. $(\mathbf{M}, s) \not\models ?_i Q^i$, *for any* s *and* $\mathbf{M}$.

Proof. It is easy to see that (in S5) the formula $(\langle i\rangle Q^i \wedge \langle i\rangle \neg Q^i)$ cannot be true in any state. It is impossible to satisfy $\langle i\rangle Q^i$ together with $\langle i\rangle \neg Q^i$ in one state. This follows from Proposition 1 and Fact 1. □

No matter whether Q^i is askable in $(\mathbf{M}, s)$ or not, the question $?_i Q^i$ is not askable in $(\mathbf{M}, s)$. In systems weaker than S5, it is possible to have a non-askable question Q (by i) in a state and, simultaneously, the question *Is Q askable (by i)?* is askable (by i) there.

2.5 Questions about knowledge

Questions about knowledge are in a similar position. An agent cannot ask about his or her own knowledge. Questions $?_i([i]\varphi)$ and $?_i(\langle i\rangle\varphi)$ are not askable. For example, $(\mathbf{M}, s) \models ?_i([i]\varphi)$ requires that both formulas $\langle i\rangle\langle i\rangle\neg\varphi$ and $\langle i\rangle[i]\varphi$ are true in $(\mathbf{M}, s)$. This is not possible in S5.[13]

On the contrary, questions about knowledge of some other agent are important in our setting. Teachers very often ask questions that are not askable for them because they know the answer. Their questions are more about the addressee's knowledge. A t(eacher) wants primarily obtain an answer to a question $?_t([i]\varphi)$, i.e., whether the student (agent) i knows φ.

2.6 Askability and belief

We use the concept of *askability* in the framework of modal logic S5, which is often considered as 'the logic' of knowledge. The operator $[i]$ is also (informally) interpreted as 'knowledge'. However, we have to be careful in using *askability* in other formal systems. Let us consider the following example where $[i]$ can be (informally) interpreted as 'agent i believes that ...':

[10] The concept of *informative formula* is a part of the dynamic extension of erotetic epistemic logic, cf. [7, pp. 91–92].

[11] Igor Sedlár drew attention to it.

[12] See the discussion on safe questions on page 8.

[13] In multi-agent setting, the question about common knowledge $?_i C_G\varphi$ is not askable for $i \in G$ either.

Suppose an agent i has the following beliefs $[i]\varphi_1, \dots, [i]\varphi_n$ in a state. If someone says *One of your beliefs is false*, it is natural for i to ask *Which one is it?*, but the following formalization of this question $?_i\{\neg\varphi_1, \dots, \neg\varphi_n\}$ is not askable in that state.[14]

2.7 Finite sets of direct answers

Our intention to provide complex information on the ignorance and expectations of an agent posing a question led us to a finite version of set-of-answers methodology, i.e., we work with questions that have finite sets of direct answers. Critics of this approach have pointed out at the impossibility of such a representation to model many 'reasonable' questions. For example, a question *Which natural number is greater than 5?* requests an answer from an infinite but well defined set of answers (natural number(s) greater than 5). Moreover, agents often ask questions without being aware of all possible direct answers (*Where is Bond?*). The original idea was to model a communication of machine-like agents that use finite databases. Finite sets of direct answers makes it possible to express some condition by (finite) formulas, see Definitions 3 and 4 that are based on the context condition (Definition 1), and Definitions 5 and 6 that define answerhood conditions. Answerhood conditions require just a slight change if we want to generalize askability for infinite questions:

> A question Q^i is *(partially) answered* in $(\mathbf{M}, s)$ for an agent i iff there is $\alpha \in dQ^i$ such that $(\mathbf{M}, s) \models [i](\neg)\alpha$.

The context condition of infinite questions remains problematic. In [7], inspired by IEL, we defined the context condition by applying the concept of *prospective presupposition*. In some cases, however, there can be a problem with non-existence of prospective presuppositions, cf. [13, Section 4.3.3.].

3 Erotetic implication

Perhaps the most important part of erotetic logic is the study of erotetic inferences. In this section, we are going to present what it means that one question *entails* another one. On the account presented here, erotetic entailment is based on (standard) implication.

Questions are formulas of the language $\mathcal{L}^{KQ}_{cpl}$, so are formulas of the form $Q^i_1 \to Q^j_2$, for agents i and j. Semantically, $(\mathbf{M}, s) \models Q^i_1 \to Q^j_2$ depends on the epistemic state $(\mathbf{M}, s)$ for both agents. However, there are no valid formulas $Q^i_1 \to Q^j_2$ for $i \neq j$. Since our interest is primarily focused on valid inferential structures, we will define erotetic implication and entailment relative to one agent.

[14]Igor Sedlár put forward this example.

Definition 7. *1. A question Q_1^i* implies *a question Q_2^i in $(\mathbf{M}, s)$ iff askability of Q_1 (in $(\mathbf{M}, s)$, by i) implies askability of Q_2 (in $(\mathbf{M}, s)$, by i), i.e., $(\mathbf{M}, s) \models Q_1^i \to Q_2^i$.*

2. We say that a question Q_1 entails *a question Q_2 iff the formula $Q_1^i \to Q_2^i$ is valid for any agent i.*

We mentioned that questions $?_i\alpha$ and $?_i\neg\alpha$ have the same askability conditions. The equivalence of both questions is a theorem in our system. Let us omit the index i in the following examples.

Fact 3. *Both $(?\alpha \to ?\neg\alpha)$ and $(?\neg\alpha \to ?\alpha)$ are valid formulas.*

Whenever Q is askable by an agent, then every question implied by Q is also askable by the agent. The (minimal) afterset substructure required by an implied question must be a substructure of that required by an implying question. The question

What is Peter: a lawyer or an economist?

implies

Is Peter a lawyer?

as well as

Is Peter an economist?

This can be generalized:

Fact 4. *$?\{\alpha_1, \ldots, \alpha_n\} \to ?\alpha_j$ is valid, for each $j \in \{1, \ldots, n\}$.*

The following example shows the special position of conjunctive questions in implications.

Fact 5. *The following formulas are valid:*

1. $?|\alpha_1, \ldots, \alpha_n| \to ?\alpha_j$, *for each* $j \in \{1, \ldots, n\}$
2. $?|\alpha, \beta| \to ?(\alpha \circ \beta)$, *where $\circ$ is any truth-functional constant*
3. $?|\alpha, \beta| \to ?\{\alpha, \beta, (\neg\alpha \land \neg\beta)\}$
4. $?|\alpha_1, \ldots, \alpha_n| \to ?|\beta_1, \ldots, \beta_j|$, *where* $\{\beta_1, \ldots, \beta_j\} \subseteq \{\alpha_1, \ldots, \alpha_n\}$[15]

It is easy to check that $?|p, q| \to ?\{p, q\}$ is not valid. There is a problem with the context condition; a risky question $?\{p, q\}$ requires the validity of the disjunction $(p \lor q)$ in the afterset.[16] The other direction $(?\{p, q\} \to ?|p, q|)$ is not valid either. If $(\mathbf{M}, s) \models ?\{p, q\}$, then the afterset structure need not satisfy $?|p, q|$. The next example shows the influence of the context condition again.

[15] Mariusz Urbański pointed out this generalization.

[16] If the context condition is canceled (and we keep non-triviality as well as admissibility, Definition 1), then this implication will be valid.

Example 1. *The following formulas are not valid:*

1. $?\{\alpha, \beta\} \to ?\{\neg\alpha, \neg\beta\}$
2. $?\{\neg\alpha, \neg\beta\} \to ?\{\alpha, \beta\}$
3. $?\{\alpha, \beta, \gamma\} \to ?\{\alpha, \beta\}$

The question $?_i\{\alpha, \beta, \gamma\}$ requires $[i](\alpha \vee \beta \vee \gamma)$, but $?_i\{\alpha, \beta\}$ requires 'only' $[i](\alpha \vee \beta)$, which can fail in the structure sufficient for the askability of the first question.

Erotetic implication has the expected property—transitivity:

Fact 6. *If* $(\mathbf{M}, s) \models Q_1^i \to Q_2^i$ *and* $(\mathbf{M}, s) \models Q_2^i \to Q_3^i$*, then* $(\mathbf{M}, s) \models Q_1^i \to Q_3^i$ *for any* $(\mathbf{M}, s)$*.*

The safeness in a state is transferred by implication. Compare also Fact 5.

Proposition 2. *If* Q_1^i *is a safe question (or, at least, safe in the state* $(\mathbf{M}, s)$*) and the formula* $(Q_1^i \to Q_2^i)$ *is valid (or, at least, satisfied in* $(\mathbf{M}, s)$*), then* Q_2^i *is safe in the state* $(\mathbf{M}, s)$*.*

Proof. Let us suppose that Q_1^i is safe and $(Q_1^i \to Q_2^i)$ is valid. If Q_1^i is safe (Definition 3), then it is safe in any epistemic state (Definition 4). Thus, for any model $\mathbf{M}$ and state s, if $(\mathbf{M}, s) \models Q_1^i$, then $(\mathbf{M}, s) \models Q_2^i$, because of the validity of $(Q_1^i \to Q_2^i)$. Since Q_2^i is askable in $(\mathbf{M}, s)$, it is also safe in $(\mathbf{M}, s)$ (condition 3 in Definition 1). □

3.1 Answerhood conditions and erotetic implication

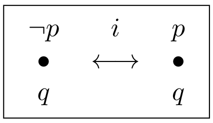
In this subsection, we want to explain the relationship between (complete) answerhood and partial answerhood with respect to erotetic implication.

The following proposition says, perhaps in a bit complicated way, that if an agent can partially answer a question Q, then she can (completely) answer a yes-no question based on the set dQ.

Proposition 3. *If a question* Q *is partially answered in* $(\mathbf{M}, s)$ *for an agent* i*, then there is a formula* φ *such that* $Q^i \to ?_i\varphi$ *is valid and the yes-no question* $?_i\varphi$ *is answered in* $(\mathbf{M}, s)$ *for* i*.*

Proof. If the admissibility condition fails, there is a direct answer which is not considered as possible, i.e., there is $\alpha \in dQ^i$ such that $(\mathbf{M}, s) \not\models \langle i \rangle \alpha$, equivalently, $(\mathbf{M}, s) \models [i]\neg\alpha$. This means that i can answer the question $?\alpha$ in $(\mathbf{M}, s)$, for some $\alpha \in dQ^i$. Using Fact 4 we finish the proof. □

Let us suppose that a question Q is (completely) answered for i. Does it mean that Q is partially answered (for i)? Unfortunately not, see the following model (for i):



In the structure, the question $?_i\{p, q\}$ is answered (in both states) because the agent i knows q, but it is not partially answered; the agent does not know either $\neg p$ or $\neg q$, both p and q are still possible.

The interplay of answerhood conditions depends, generally, on epistemic structure. Questions with *pairs of mutually exclusive direct answers* behave properly.[17] Yes-no questions as well as conjunctive questions are examples from this class. Their sets of direct answers satisfy a more strict condition, they have *mutually exclusive* direct answers—the 'truth' of a direct answer (in a state) means that no other direct answer is satisfied there.

Both kinds of mutual exclusiveness are of a semantic nature, they can be caused by a state in a model. Recall Carl's question *Who has the Joker: Anne, or Bond?* with the context, which is very important here. The context does not admit a substructure similar to the previous one.

Mutual exclusiveness is not preserved by implication. In particular, the answers to the question $?_i\{\alpha \wedge \beta, \alpha \wedge \neg\beta, \neg\alpha \wedge \beta\}$ are mutually exclusive, but this question implies $?_i\{\alpha, \beta\}$, which need not be in this class.

Partial answerhood of a question Q^i (with at least pairs of mutual exclusive direct answers) in some state is equivalent to the existence of a yes-no question, which is answered in that state and implied by Q^i.

Proposition 4. *If a question Q^i is safe (for i) in $(\mathbf{M}, s)$ and dQ^i consists of pairs of mutually exclusive formulas, then the following conditions are equivalent:*

1. *The question Q^i is non-askable in $(\mathbf{M}, s)$, i.e., $(\mathbf{M}, s) \not\models Q^i$.*
2. *The question Q^i is partially answered in $(\mathbf{M}, s)$ (for i).*
3. *There is a formula φ such that the question $?_i\varphi$ is answered in $(\mathbf{M}, s)$ (for i) and $(Q^i \rightarrow ?_i\varphi)$ is valid.*

Proof. (2⇒1) is clear and (2⇒3) comes from Proposition 3.

(1⇒2) If $(\mathbf{M}, s) \not\models Q^i$, then there are three possibilities: Q^i is answered or partially answered or the context condition is not satisfied. The last one is impossible because of the safeness of Q^i in $(\mathbf{M}, s)$. If Q^i is answered in $(\mathbf{M}, s)$ for i, then it is partially answered because dQ^i consists of pairs of mutually exclusive formulas.

(3⇒1) Let us suppose that there is a formula φ such that the question $?_i\varphi$ is answered in $(\mathbf{M}, s)$ for i. From $(Q^i \rightarrow ?_i\varphi)$ we know that if $(\mathbf{M}, s) \not\models ?_i\varphi$, then $(\mathbf{M}, s) \not\models Q^i$. □

[17] *Pairs of mutually exclusive direct answers* means that for each direct answer there is another one such that both of them cannot be true in one state.

From the validity of $(Q^i \to ?_i\varphi)$ we know that the non-askability of $?_i\varphi$ implies non-askability of Q^i and, therefore, φ (as well as $\neg\varphi$) implies either some α or $\neg\alpha$, for $\alpha \in dQ^i$.

3.2 Sets of implied yes-no questions

If questions are in the implicational relationship, the transmission of askability conditions from an implying question to the implied one is justified. An implied question is understood as 'less complex' in its requirements posed on the afterset substructure. Let us recall Fact 4 where $(Q \to ?\alpha)$ is valid for each $\alpha \in dQ$.[18] Because of the finiteness of dQ we can write

$$Q \to \bigwedge_{\alpha \in dQ} (?\alpha)$$

If we consider the set of yes-no questions $\Phi = \{?\alpha_1, \ldots, ?\alpha_n\}$, then the non-askability of some $?\alpha$ means that the answer is either α or $\neg\alpha$. In the first case, Q is answered as well, in the second one, Q is partially answered. This can be understood as a form of 'sufficiency' condition of the set Φ:

> The answerability of some $Q_j \in \Phi$ implies (at least) the partial answerability of the initial question Q.

It means that Φ does not include 'useless' questions. Moreover, we receive one more property, Φ is 'complete' in a way:

> If Q is partially answered, then there must be a question $Q_j \in \Phi$ that is answered.

The transfer from Q to Φ can be called *reducibility* of Q to Φ. An initial question Q is reducible to a set of yes-no questions Φ. In contradistinction with IEL, which inspired this idea, our reducibility is based purely on implication.

On the one hand, each question $?\alpha$ is a yes-no question with some good properties. Yes-no questions are safe questions with mutually exclusive direct answers. On the other hand, they may be 'worse' in the description of an agent's knowledge/ignorance structure than the initial question Q.

Fact 5 gives a similar result for conjunctive questions. The set of yes-no questions can be formed by constituents of their direct answers:

$$?|\alpha_1, \ldots, \alpha_n| \to \bigwedge_{j=1}^{n} (?\alpha_j)$$

In this case, we arrive at really less complex questions. Having a partial answer to $?|\alpha_1, \ldots, \alpha_n|$ does not give either an answer or a partial answer to some $?\alpha_j$. It could be useful in some cases. An agent can ask questions from the set $\Phi = \{?\alpha_1, \ldots, ?\alpha_n\}$ and complete her knowledge step by step. The most important

[18] Let us omit the index i in this subsection.

property is that the set Φ does not include useless questions. Generally speaking, in some communication processes it is useful to conceal some knowledge or ignorance of a questioner—a criminal investigation is a nice example. An agent can ask questions from the set Φ without completely revealing her knowledge structure. Asking a conjunctive question $?|\alpha_1, \ldots, \alpha_n|$ publicly, everybody is informed that the agent-questioner does not know anything with respect to $\alpha_1, \ldots, \alpha_n$.

4 Context (generally)

The term *context* has been used in two meanings. The first one, *context condition* in Definition 1, is more bounded. It is a presupposition of a question, based purely on the direct answers. It forms a border for agent's acceptable answers.

The second meaning was demonstrated in the introductory example with the Joker card. This context was based on additional information (*Just one Joker is distributed among the agents*) that complemented the context condition. This kind of context can be formalized as a set of auxiliary formula(s). The idea of posing a question with respect to a set of auxiliary formulas will be discussed here. An inspiration from IEL will be apparent.

Relationships between a question and a set of formulas are going to be demonstrated as a generalization of two types of questions: *conditional questions* and *hypothetical questions* (see [13]).

4.1 Conditional and hypothetical askability

Let us consider the following question:

Did you stop smoking?

At first sight, it is a yes-no question, but seeing both answers it seems that there is something more that is presupposed:

- *Yes, I did* can mean *I had smoked and stopped.*
- *No, I didn't* can mean *I smoked and go on.*

Both of them presuppose the smoking in the past. Such a question is an example of a *conditional yes-no question* with a formalization $?_i\{(p \wedge q), (p \wedge \neg q)\}$ where p formalizes the smoking in the past. Generally, *conditional questions* are of the form

$$?_i\{\alpha \wedge \beta_1, \alpha \wedge \beta_2, \ldots, \alpha \wedge \beta_n\}$$

and consist of two parts: a conditional part (context) and a query part. The askability of a conditional question in a state s requires the validity of α in each accessible state. This is caused by the context condition and it means that an agent 'knows' α in s.

The concept of an askable (finite) conditional question can be generalized to a concept of a question askable with respect to an auxiliary set of formulas:

Definition 8 (Conditional askability)**.** *A question Q^i is askable (by an agent i) in $(\mathbf{M}, s)$ with respect to a set of formulas Γ iff $(\mathbf{M}, s) \models \{[i]\gamma \mid \gamma \in \Gamma\}$ and $(\mathbf{M}, s) \models Q^i$.*

A conditional question $?_i\{\alpha \wedge \beta_1, \alpha \wedge \beta_2, \ldots, \alpha \wedge \beta_n\}$ is askable in s if and only if $?_i\{\beta_1, \ldots, \beta_n\}$ is askable there with respect to the auxiliary set of formulas (knowledge base) $\{\alpha\}$.

Carl's question from our introductory example

Who has the Joker: Anne, or Bond?

can be understood as a question $?_c\{p, q\}$ with respect to $\{\neg p \vee \neg q\}$.

The other type of questions we are going to study is *hypothetical question*, e.g.:

If you open the door, will you see a bedroom?

which can be formalized by $?_i\{(p \to q), (p \to \neg q)\}$. A general schema of a hypothetical question is:

$$?_i\{\alpha \to \beta_1, \ldots, \alpha \to \beta_n\}$$

The askability of such questions can be understood as based on an agent's hypothetical knowledge. Our interpretation is: if α is known, then it is to be decided whether β_1, or β_2, etc. Using a generalization similar to Definition 8 we obtain:

Definition 9 (Hypothetical askability)**.** *A question Q^i is askable (by an agent i) in $(\mathbf{M}, s)$ with respect to a set of hypotheses Γ iff $(\mathbf{M}, s) \models \{[i]\gamma \mid \gamma \in \Gamma\}$ implies $(\mathbf{M}, s) \models Q^i$.*

4.2 Reasoning with context

The concepts of askability with respect to sets of auxiliary formulas, introduced in Definitions 8 and 9, can be used to display and emphasize the role of context in reasoning. An agent asking

Q_1: *Which faculty did Peter graduate from: a faculty of law or a faculty of economics?*

can be satisfied by the answer

He is a lawyer.

even if she did not ask

Q_2: *What is Peter: a lawyer or an economist?*

The connection between both questions could be established by the following knowledge base Γ:

Someone is a graduate of a faculty of law iff he or she is a lawyer.
Someone is a graduate of a faculty of economics iff he or she is an economist.

To be able to speak about the properties of these concepts we have to introduce some abbreviations. First, let us write $(\mathbf{M}, s) \models [i]\Gamma$ instead of $(\mathbf{M}, s) \models \{[i]\gamma \mid \gamma \in \Gamma\}$. Second, whenever we want to express that a question Q^i is askable by an agent i in $(\mathbf{M}, s)$ with respect to a set of formulas Γ (Definition 8), we will write

$$(\mathbf{M}, s) \models ([i]\Gamma \bowtie Q^i)$$

This abbreviation will help us to speak about (general) *epistemic erotetic implication* (shortly *e-e-implication*). In the next definition we are going to define that a question Q_2^i is e-e-implied by Q_1^i with respect to Γ in a state $(\mathbf{M}, s)$, which will be formalized as

$$(\mathbf{M}, s) \models ([i]\Gamma \bowtie Q_1^i) \Rightarrow Q_2^i$$

Definition 10 (e-e-implication)**.** $(\mathbf{M}, s) \models ([i]\Gamma \bowtie Q_1^i) \Rightarrow Q_2^i$ *iff* $(\mathbf{M}, s) \models ([i]\Gamma \bowtie Q_1^i)$ *implies* $(\mathbf{M}, s) \models Q_2^i$.

The last abbreviation we are going to introduce expresses that a question Q is askable in a state $(\mathbf{M}, s)$ with respect to a set of hypotheses Γ (cf. Definition 9):

$$(\mathbf{M}, s) \models [i]\Gamma \Rightarrow Q^i$$

Now we can list some properties in an readable manner. Most of them are apparent and expected.

A question askable with respect to (a context) Γ is askable with respect to a smaller context:

Fact 7. *If* $(\mathbf{M}, s) \models ([i]\Gamma \bowtie Q^i)$*, then* $(\mathbf{M}, s) \models ([i]\Delta \bowtie Q^i)$ *for every* $\Delta \subseteq \Gamma$*.*

This property is no surprise. We have to keep in mind that an epistemic state $(\mathbf{M}, s)$ and its surrounding sR_i fully describe conditions for askability of a question (by an agent). However, this description is implicit. Conditional askability explicitly display some important agent's knowledge true in the state and (possibly) important for the askability of a question. Clearly, it need not be $(\mathbf{M}, s) \models ([i]\Delta \bowtie Q^i)$, for any set of formulas Δ. As an easy corollary we obtain:

Fact 8. *If* $(\mathbf{M}, s) \models ([i]\Gamma \bowtie Q^i)$*, then* $(\mathbf{M}, s) \models Q^i$*.*

Let us notice that a conditional question implies its query part:

Fact 9. $?_i\{\alpha \wedge \beta_1, \ldots, \alpha \wedge \beta_n\} \rightarrow ?_i\{\beta_1, \ldots, \beta_n\}$.

This is not true for hypothetical questions.[19] We have just described the 'local' behavior (in a state). From the 'global' point of view, it need not be valid that $(\Delta \bowtie Q)$, even if $(\Gamma \bowtie Q)$ is valid, for $\Delta \subset \Gamma$.

Erotetic epistemic implication is 'locally monotonic':

[19] $?_i\{\alpha \rightarrow \beta_1, \ldots, \alpha \rightarrow \beta_n\}$ does not imply $?_i\{\beta_1, \ldots, \beta_n\}$.

Proposition 5. *If* $(\mathbf{M}, s) \models ([i]\Gamma \bowtie Q_1^i) \Rightarrow Q_2^i$*, then* $(\mathbf{M}, s) \models ([i](\Gamma \cup \Delta) \bowtie Q_1^i) \Rightarrow Q_2^i$*, for any sets of formulas* Γ, Δ.

Proof. Let us suppose $(\mathbf{M}, s) \models ([i]\Gamma \bowtie Q_1^i) \Rightarrow Q_2^i$ and $(\mathbf{M}, s) \models ([i](\Gamma \cup \Delta) \bowtie Q_1^i)$, then we obtain $(\mathbf{M}, s) \models Q_2^i$. Whenever $(\mathbf{M}, s) \not\models Q_2^i$, then either $(\mathbf{M}, s) \not\models Q_1^i$ or $(\mathbf{M}, s) \not\models [i]\Gamma$, but it is impossible. □

E-e-implication is also 'globally monotonic', which follows from the previous proposition.

Fact 10. *If* $([i]\Gamma \bowtie Q_1^i) \Rightarrow Q_2^i$ *is valid, then* $([i](\Gamma \cup \Delta) \bowtie Q_1^i) \Rightarrow Q_2^i$ *is valid, for any sets of formulas* Γ, Δ.

The following proposition speaks about transitivity of e-e-implication with possibly different contexts.

Proposition 6. *If* $(\mathbf{M}, s) \models ([i]\Gamma \bowtie Q_1^i) \Rightarrow Q_2^i$ *and* $(\mathbf{M}, s) \models ([i]\Delta \bowtie Q_2^i) \Rightarrow Q_3^i$*, then* $(\mathbf{M}, s) \models ([i](\Gamma \cup \Delta) \bowtie Q_1^i) \Rightarrow Q_3^i$.

Proof. Let $(\mathbf{M}, s) \models ([i](\Gamma \cup \Delta) \bowtie Q_1^i)$. Then, from Proposition 5, $(\mathbf{M}, s) \models ([i](\Gamma \cup \Delta) \bowtie Q_1^i) \Rightarrow Q_2^i$ as well as $(\mathbf{M}, s) \models ([i](\Gamma \cup \Delta) \bowtie Q_2^i) \Rightarrow Q_3^i$. Thus, $(\mathbf{M}, s) \models ([i](\Gamma \cup \Delta) \bowtie Q_1^i) \Rightarrow Q_3^i$ □

Conditional askability is transferred by e-e-implication.

Proposition 7. *If* $(\mathbf{M}, s) \models ([i]\Gamma \bowtie Q_1^i)$ *and* $(\mathbf{M}, s) \models ([i]\Gamma \bowtie Q_1^i) \Rightarrow Q_2^i$*, then* $(\mathbf{M}, s) \models ([i]\Gamma \bowtie Q_2^i)$.

Proof. We have to prove that $(\mathbf{M}, s) \models \{[i]\gamma \mid \gamma \in \Gamma\}$ as well as $(\mathbf{M}, s) \models Q_2$. The former is from $(\mathbf{M}, s) \models ([i]\Gamma \bowtie Q_1^i)$ and the latter follows from $(\mathbf{M}, s) \models ([i]\Gamma \bowtie Q_1^i) \Rightarrow Q_2^i$ together with $(\mathbf{M}, s) \models Q_1^i$, which is also from $(\mathbf{M}, s) \models ([i]\Gamma \bowtie Q_1^i)$. □

Hypothetical askability is also transferred by e-e-implication.

Proposition 8. *If* $(\mathbf{M}, s) \models [i]\Gamma \Rightarrow Q_1^i$ *and* $(\mathbf{M}, s) \models ([i]\Gamma \bowtie Q_1^i) \Rightarrow Q_2^i$*, then* $(\mathbf{M}, s) \models [i]\Gamma \Rightarrow Q_2^i$.

Proof. Let us suppose $(\mathbf{M}, s) \not\models [i]\Gamma \Rightarrow Q_2^i$, i.e., $(\mathbf{M}, s) \models [i]\Gamma$ and $(\mathbf{M}, s) \not\models Q_2^i$. From $(\mathbf{M}, s) \models [i]\Gamma$ and $(\mathbf{M}, s) \models [i]\Gamma \Rightarrow Q_1^i$ we get $(\mathbf{M}, s) \models Q_1^i$ and $(\mathbf{M}, s) \models ([i]\Gamma \bowtie Q_1^i)$. Because of $(\mathbf{M}, s) \models ([i]\Gamma \bowtie Q_1^i) \Rightarrow Q_2^i$ we obtain $(\mathbf{M}, s) \models Q_2^i$, which is a contradiction. □

The following Fact is a variant of Propositions 7 and 8 where (pure) erotetic implication is used.

Fact 11.

- *If* $(\mathbf{M}, s) \models ([i]\Gamma \bowtie Q_1^i)$ *and* $(\mathbf{M}, s) \models (Q_1^i \to Q_2^i)$*, then* $(\mathbf{M}, s) \models ([i]\Gamma \bowtie Q_2^i)$.
- *If* $(\mathbf{M}, s) \models [i]\Gamma \Rightarrow Q_1^i$ *and* $(\mathbf{M}, s) \models (Q_1^i \to Q_2^i)$*, then* $(\mathbf{M}, s) \models [i]\Gamma \Rightarrow Q_2^i$.

In the previous subsection, conditional and hypothetical askability was introduced as inspired by conditional and hypothetical questions. Because of context condition, these concepts are directly related. We can define conditional askability for any conditional question and the same holds for hypothetical questions and hypothetical askability. The other direction is also valid if the context is finite:

Fact 12.
- *If* Γ *is finite and* $(\mathbf{M}, s) \models ([i]\Gamma \bowtie Q^i)$*, then there is a question* Q^i_* *such that* $(\mathbf{M}, s) \models Q^i_*$*.*
- *If* Γ *is finite and* $(\mathbf{M}, s) \models [i]\Gamma \Rightarrow Q^i$*, then there is a question* Q^i_* *such that* $(\mathbf{M}, s) \models Q^i_*$*.*

5 Conclusion

In this paper we presented epistemic logic with questions. The introduced system, *erotetic epistemic logic* (EEL), is a logic of questions with all attributes that are usually required in this branch of logic. In the framework of epistemic logic, we introduced question formalization, semantics of questions, answerhood conditions, and reasoning with questions.

Epistemic logic is a static description of knowledge, ignorance, and epistemic possibilities. So is EEL. Nonetheless, posing and answering of questions is understood as a dialog-style communication. The presented system is friendly to dynamization. In our previous work, cf. [8, 9, 7], we decided to use multi-agent version of epistemic logic, which made it possible to extend answerhood conditions to groups of agents. Commonly known answer to a question was necessary for to say that the question is answered in a group. Distributed knowledge played the role of implicit answerability—there could be a question, which is askable by the group (i.e., by every member), but the group can reach an answer just by communication. The answer is hidden for every member but reachable. We modeled communication in a group of agents with the help of *public announcement logic*. Questions behave properly in this framework. Publicly asked questions are successful formulas and, thus, they become commonly known. Moreover, if a question is askable in a state, then this question is a successful update there—it becomes true in the updated state. The 'solvability' of a group question is then based on (finite list of) announcements of agents' knowledge. Agents in the group pool their knowledge together. This approach is open to an employment of algorithms similar to *erotetic search scenarios* [10, 15]. This is the idea we are going to develop.

Nothing surprising in this branch of logic that EEL has some points that can be subjected to discussion. Our motivation was to prepare an erotetic background useful for various dynamic extensions of epistemic logic S5 where the asking and answering strategy is machine friendly and conducted by (variants of) epistemic erotetic implication. Simultaneously, we believe that EEL can be an inspiration for other epistemic-like systems with questions.

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